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## Equity Optimization Issues III: Insignificant Alphas, Heterogeneous Errors

by

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**Abstract**

Insignificant alphas and heterogeneous estimation error are two issues associated with performance limitations in mean variance equity portfolio optimization.

This essay is the third in a series on issues and proposals for improving the investment value of equity portfolio optimization.<sup>1</sup> For discussion efficiency, the reader is referred to the two previous newsletters on the same theme for definitions and basic concepts.<sup>2</sup>

### **Equity Optimization Practices**

Markowitz (1959) mean-variance (MV) efficiency has been a standard for theoretical finance and investment management for nearly fifty years. But the MV optimization procedure is a numerical algorithm that does not consider uncertainty in investment data. The lack of a statistical context leads to severe performance limitations. MV equity portfolio optimization also includes a legacy of ad hoc practices to cosmetically control the character of the optimized portfolio.

Resampled Efficiency™ (RE) optimization is designed to manage estimation error appropriately in a MV optimization. It is the only provably investment effective portfolio optimizer in the world today. Unfortunately, many cosmetic procedures for controlling MV equity portfolio optimization are self-defeating in the context of an estimation error sensitive optimizer.<sup>3</sup> Prior newsletters focused on avoiding some of these self-defeating practices. This newsletter considers two additional issues associated with performance limitations in MV equity portfolio optimization and their resolutions. These are: 1) Insignificant alphas. 2) Heterogeneous estimation error.<sup>4</sup>

#### **I. Insignificant Alphas**

Equity portfolios are typically optimized relative to a stock index with many stocks. The index stock universe may include as many as 5000 stocks for domestic managers and many more for international managers. In large stock universe equity optimization, a high proportion, often-large majorities, of stocks are statistically insignificant.<sup>5</sup>

The proper goal of equity portfolio optimization is to over- and under-weight investment significant stocks. Allowing an optimizer to rattle around with a large proportion of statistically insignificant information exposes the optimized portfolio to enormous estimation error and is a recipe for investment dysfunction. In this case, equity portfolio optimization is little more than optimization in white noise. But equity optimization of only significant alpha stocks may lead to under representing large segments of the index and exposing the optimized portfolio to substantial tracking error risk. Managing tracking error risk is often the major reason for the existence of ad hoc constraints and other procedures in current investment practice.

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<sup>1</sup> Michaud and Michaud (2004, 2005).

<sup>2</sup> These and other newsletter articles are available at [www.newfrontieradvisors.com](http://www.newfrontieradvisors.com).

<sup>3</sup> Richard Michaud and Robert Michaud are co-inventors: U.S. Patent awarded December 1999, worldwide patents pending. New Frontier Advisors, LLC is the exclusive worldwide licensee.

<sup>4</sup> Alpha inputs into an equity optimization are formally systematic risk-adjusted forecast returns. However, there are many variations used in practice. Intuitively, alpha represents the forecast return associated with an active investment strategy risk-adjusted.

<sup>5</sup> A simple criterion is the alpha/residual risk ratio and test for t-stat significance. One key issue, whether the alphas have been appropriately scaled, is addressed further in Michaud and Michaud (2005).

### **Zero-Alpha Composite Asset**

We present a simple solution for solving the insignificant alpha estimation error optimization problem. Segment the stocks in the index universe into significant and non-significant alphas. Form the insignificant alpha stocks into an index-weighted composite portfolio and assign a zero alpha.<sup>6</sup> Define the optimization universe as the significant alpha stocks and the composite as a single additional asset. For example, suppose 1000 stocks in the index, 900 of which have insignificant alphas. The proposal is to base the optimization on a universe of 101 assets, 100 significant alpha stocks and 1 composite index-weighted asset of the 900 insignificant assets.<sup>7</sup>

All stocks in the universe are likely to have some weight in such an RE equity-optimized portfolio: significant stocks due to their investment value and insignificant alpha stocks due to their index weights. Investability and transaction cost considerations can be imposed to reduce the securities to a convenient number and define investable weights in the investment portfolio. Note that the composite asset framework manages tracking error naturally and makes many of the ad hoc procedures commonly used to define a MV optimized portfolio obsolete. This framework holds great promise in appropriately managing estimation error, simplifying the equity portfolio optimization process, and dramatically reducing computing time especially for large stock universes.

### **II. Heterogeneous Estimation Error**

In equity optimization, the range of stock capitalization typically varies widely. In many cases 10% of the largest stocks may represent 80% or more of the total capitalization of the index. Cap size is an important factor in optimization design due to its relationship with estimation error. Smaller cap size stocks have more estimation error all other things the same. Smaller stocks are not as well followed by analysts, have less oversight by regulators, and have less liquidity. Because of this, some estimates of risk and return may not be as reliable as others.

Information heterogeneity presents important difficulties for equity portfolio optimization. Fundamentally, MV optimization is equivalent to generalized least-squares estimation with equality and inequality constraints and unknown covariance matrix. The typically wide variation in the cap size distribution in equity portfolio optimization implies substantial estimation error heterogeneity in optimization inputs and risk (covariance matrix) estimation.<sup>8</sup>

### **Estimation Error Weighted RE Optimization**

We introduce a new procedure – estimation error weighted RE optimization – for dealing with estimation error heterogeneity in equity portfolio optimization. One of the simplest econometric methods for dealing with heterogeneous or heteroscedastic error term variance is weighted least squares. In this case we use relative cap size as the basis of the

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<sup>6</sup> The significant alpha stocks should index-weight sum to zero. Residual risk is defined by the risk model.

<sup>7</sup> This option will be made available in NFA's equity optimization technology in the near future.

<sup>8</sup> Generally, asset allocation studies involve liquid, well diversified asset classes with relatively homogeneous estimation error. However, asset allocations that include leveraged hedge funds or other alternative asset classes may be exceptions.

weighting function. Each observation is weighted by the inverse of a function of relative cap size reflecting the fact that large cap stocks are more reliable and need to play a greater role in the optimization estimation process than small cap stocks all other things the same.<sup>9</sup>

Pure capitalization weighting is unlikely to be a useful model of estimation error heterogeneity for many capital market indices. This is because very large stocks may overwhelm the estimation error weighted optimization process. However, a monotonic function of cap size, such as the log of capitalization, properly calibrated, is often convenient and appropriate. Estimation-error weighted RE optimization represents a fundamentally new and powerful tool for dealing with estimation error heterogeneity in equity portfolio optimization in many cases in investment practice.<sup>10</sup>

### Conclusions

Many ad hoc procedures are part of classical MV equity optimization in practice. Because MV optimization is numerical rather than statistical, these procedures have the effect of sculpturing a portfolio according to preconceived characteristics. Meaningful optimization of investment information is not possible with a numerical procedure.

Because RE optimization is sensitive to estimation error many of the traditional procedures of MV optimization are unnecessary and even counterproductive. In this third of a series essay we have presented proposals for better understanding and improving the investment value of statistically sensitive optimized portfolios. The zero-alpha composite portfolio technique obsoletes many of the ad hoc procedures for constraining optimized equity portfolios and simplifies the process. Estimation error weighted RE optimization provides a means for addressing heterogeneous estimation error optimization universes common to equity portfolio optimization. Properly understood and used, RE optimization provides an investor with the possibility of greatly simplified portfolio optimization and improved performance.

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<sup>9</sup> Note that capitalization-weighted methods change the scale of the variances but leave the correlations unchanged.

<sup>10</sup> This procedure will be explored in future research and included in NFA's optimization technology in the near future.

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