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Optimization with Non-Normal Resampling

by

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Abstract

In order to retain some of the normal distribution relevance for MC optimization, we use a multivariate distribution procedure that allows for exogenous specification of skewness and kurtosis.

Markowitz mean-variance (MV) optimization promises a satisfying solution to the problem of portfolio construction. An investor determines the ideal portfolio given a level of risk averseness using the mean and covariance of the assets under consideration. Unfortunately the optimization itself is very sensitive to the inputs and typically recommends highly concentrated portfolios. The problem arises because classical MV optimization ignores the uncertainty implicit in estimates of return and risk.

Resampled Efficiency™ optimization[1] addresses this difficulty by optimizing over many possible future outcomes of the asset returns based on the given mean and covariance. The New Frontier Advisors (NFA) optimizer may perform hundreds of optimizations and averages them to produce the new efficient frontier. While not required as part of the procedure, Resampled Efficient Frontier™ (REF) optimizations often assume that the mean vector and covariance structure are (multivariate) normally distributed.

Investors may not always believe that a normal distribution assumption is appropriate. The normal distribution does not approximate all asset return distributions well. For example, hedge fund returns may often reliably exhibit consistent large degrees of non-normality. With this difficulty in mind, an investor might prefer to resample with non-normal returns to get a more accurate solution to REF optimizations.

Non-Normal REF Optimizations

Deciding to use non-normal returns in resampling creates an immediate difficulty. Which multivariate distribution should be used in place of the normal distribution? While mathematical expediency is not a good justification, the normal distribution does have the advantage of consistency with the Markowitz MV optimization framework. In order to retain some of the normal distribution's relevance for MV optimization, we use a multivariate distribution procedure that allows for exogenous specification of skewness and kurtosis.

Practically, skewness measures which side of the mean returns tend towards and how far from the mean they fall. Kurtosis measures the likelihood of tail events as well as clustering around the mean. Skewness and kurtosis are reasonable proxies for additional information about the multivariate distribution because they can be estimated from data or based on an investor's views.

NFA's newest version of the REF optimizer, Version 4.2, has a new option that allows multivariate sampling of non-normal data given skewness and kurtosis inputs in addition to the mean and covariance. The method we employ is based on the work of Fleishman (1978) and Headrick (1999). Basically, each individual asset return is generated by a cubic transformation of a standard normal random variable. The generated return distributions are functionally similar to the normal distribution.

[1] Described in R. Michaud, *Efficient Asset Management*, Oxford University Press, 1998. Resampled Efficiency is a US patented procedure, worldwide patents pending. New Frontier Advisors is the exclusive worldwide licensee.

A bit of caution is necessary. In a multivariate setting correlations and higher moments can not be chosen independently of each other. It is not possible for two assets with significantly different kurtosis to be highly correlated. In situations where the correlation matrix is incompatible with the given skewness and kurtosis, the procedure changes the correlation matrix as needed.

When resampling with non-normal distributions, certain asymptotic properties of the resampled data should be kept in mind. By construction the data asymptotically has the given mean, covariance structure, skewness and kurtosis. Each time the data is resampled, the optimizer is given the sample mean and sample covariance. Because the simulated returns are independent, the sample mean will be approximately normal. Taking the arithmetic mean reduces much of the skewness and kurtosis effect. Similarly, we expect the distribution of the covariance matrix to be close to the Wishart distribution. Resampling with non-normal returns won't significantly change the optimizer results except in the presence of significant skewness and kurtosis.

Examples Using Non-Normal Resampling

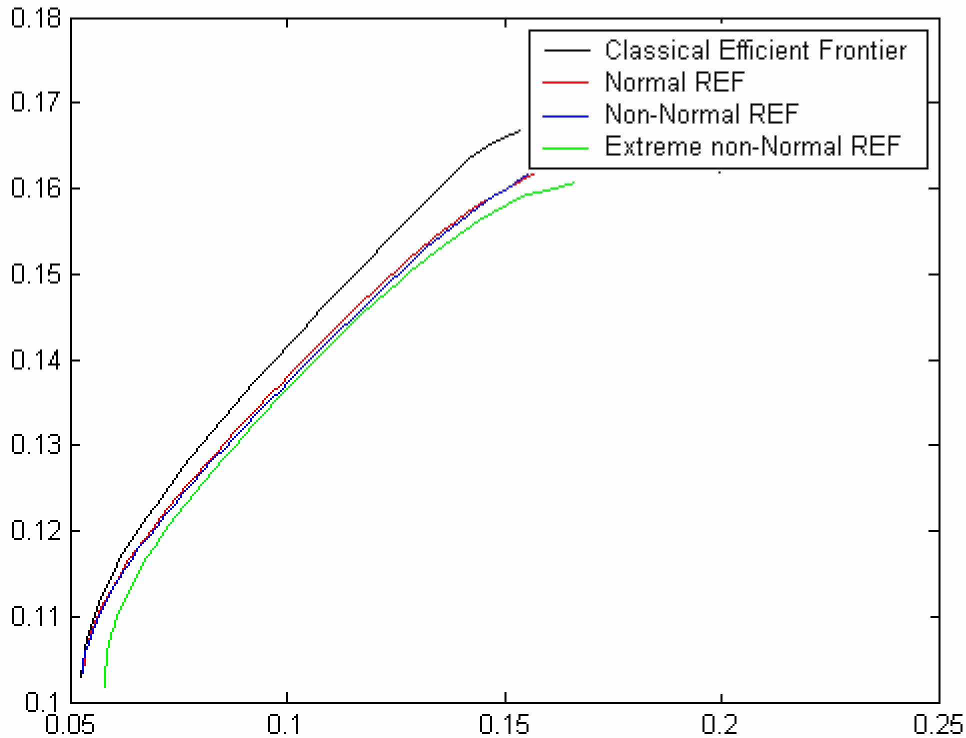
We present some results of resampling with non-normal return distributions. Ultimately there is uncertainty about high dimensional optimization problems. Unless the objective function is very simple, it is hard to have any intuition for what a solution should be. While skewness and kurtosis will have some effect on portfolios, correlations will also drive the optimization process.

In what follows we use the Michaud (1998) dataset which consists of monthly returns for eight assets between 1978 and 1995 for Canada, France, Germany, Japan, UK, US, US Bonds and Euro Bonds. We summarize the relevant input data in the following table. The table displays the mean, standard deviation, skewness and kurtosis for the monthly returns. The example of extreme non-normality gives US Bonds a skewness of 5 and kurtosis of 100.

	Mean	Standard Deviation	Skewness	Kurtosis
Canada	1.0%	5.5%	-0.4	5.6
France	1.4%	7.0%	-0.1	4.2
Germany	1.1%	6.2%	-0.1	4.0
Japan	1.4%	7.0%	0.2	3.6
UK	1.4%	6.0%	-0.1	4.0
US	1.3%	4.3%	-0.6	6.5
US Bonds	0.8%	2.0%	0.5	6.6
EU Bonds	0.8%	1.5%	0.0	5.2

Point Estimates for Monthly Returns

The following figure displays the classical efficient frontier, REF optimized allocations, REF non-normal resampling, and extreme REF non-normal resampling.



The following table summarizes the relevant data for the medium variance portfolio. Except in the extreme case for bonds, the changes in the medium variance portfolio are not very large.

	Normal REF Allocation	Non-Norm Allocation	Extreme Non-Normal Allocation
Canada	1.9%	2.4%	3.0%
France	7.5%	6.5%	6.6%
Germany	5.8%	5.0%	6.6%
Japan	14.6%	12.8%	12.4%
UK	8.7%	8.1%	5.0%
US	16.1%	19.8%	17.3%
US Bonds	14.3%	10.9%	23.2%
EU Bonds	31.2%	34.5%	25.9%

Medium Variance Portfolio

Summary and Conclusions

REF optimization corrects MV optimization's greatest deficiencies: error maximization and poorly diversified portfolios. The performance of REF allocations will also depend on the quality of estimates. The new non-normal resampling option is now available and may be useful in some special cases and for stress testing the optimization process. However, our results have found that, except in extreme cases, a REF optimized portfolio using normal return resampling generally provides an excellent approximation to optimized portfolios with non-normal skewed and kurtotic distributions.