



An Introduction to Resampled Efficiency

by

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Abstract

Resampled Efficiency provides the solution to using uncertain information in portfolio optimization.

The proper purpose of investment advice is to improve a client's portfolio in terms of maximizing return for an appropriate level of risk. Asset management techniques for optimizing the investment value of forecasts of return and risk have been available for fifty years. Markowitz provides the classic definition of optimality: a portfolio is risk-return efficient if no other portfolio has higher expected return for a given level of risk or less risk for a given level of expected return.¹ The set of all portfolios that are risk-return efficient are said to form the Markowitz efficient frontier. Markowitz also developed mathematical methods for solving the risk-return optimization problem.²

From the perspective of fifty years, it is paradoxical that many managers and advisors don't use portfolio optimization investment technology in their work. While until relatively recently optimization computer programs were difficult to use and often required special analytical ability, it is not true today. Many commercially available programs for computing Markowitz efficient portfolios require virtually no computer or analytical expertise. Yet the paradox of managers and advisors ignoring investment technology that, conceptually, uses their information optimally, remains. Are advisors not acting in their own or their client's best interests?

Advisors offer a variety of excuses for not using portfolio optimizers. Some note lack of sophistication by their competitors or clients; others indicate that issues such as taxes or client servicing have priority over risk-return optimality. However, consider what would happen if it became widely known that advisors who use portfolio optimizers provide superior investment performance. Would a good client servicing bedside manner really compete with a competitor's ability to provide superior investment performance?

Consider an alternative hypothesis: portfolio optimizers have little, if any, investment value? If true, the benign neglect of modern portfolio theory by much of the investment community, however inconsistent with the exhortations of finance textbook writers, can be rationalized.

Optimizers have been found to have a number of serious limitations in investment practice. Many sophisticated asset managers and advisors have tried to use portfolio optimization technology in their business, and ended up discarding it. One reason is that optimizers are hard to manage. Even small changes in optimization inputs often lead to large changes in optimized portfolios. Since investment information is never known with certainty, the instability of the procedure leaves users with little confidence in the results. Advisors often include many constraints in order to stabilize the optimization. The end result, however, is that you constrain the problem to be what you wanted, and it is unclear what, if anything, the optimizer contributes to portfolio investment value.

Investment intuition can be a useful guide but it should be supplemented with scientific evidence if available. In the case of the investment behavior of portfolio optimizers, the

¹ Harry Markowitz, 1952. "Portfolio Selection." *Journal of Finance* 7(1): 47-62.

² Harry Markowitz, 1956. "The Optimization of a Quadratic Function Subject to Linear Constraints." *Naval Research Logistics Quarterly* 3(1/2): 111-133.

evidence came more than twenty years ago, in a series of papers authored by two financial economists, J.D. Jobson and Bob Korkie, of the University of Alberta. They showed mathematically and statistically that optimizers have, on average, little, if any, investment value and that equal weighted portfolios are often far superior to optimized portfolios.³

Jobson and Korkie's papers provide the key to understanding the limitations of portfolio optimizers in practice. Classical optimizers assume 100% certainty in the information. But investment information is inherently uncertain. Optimizers tend to "error-maximize" investment information creating portfolios that are far too specific to a given set of inputs and consequently reflect little intuition or investment value.⁴

Resampled efficiency, described in Michaud⁵ and invented and patented by Richard Michaud and Robert Michaud,⁶ provides the solution to using uncertain information in portfolio optimization. The method is based on resampling optimization inputs. This is a Monte Carlo simulation procedure to create alternative optimization inputs that are consistent with the uncertainty in your forecasts. To illustrate, suppose you forecast a 10% return and a 20% standard deviation for the S&P500 index. You would probably not be surprised to know that the true return is 13% or 8% or that the true standard deviation is 25% or 15%. You have, of course, uncertainty in the forecasts of other assets as well. While estimate uncertainty may be relatively easy to manage with one or two assets, it becomes all but impossible when the portfolio has five or more assets or funds.

The resampling process uses your forecasts to indicate the many ways capital markets and assets may behave in the investment period. This idea is very similar to observing the behavior of a fair coin when tossed ten times. You expect to see five heads in ten tosses on average. But actual tosses of the coin may result in two or nine heads even when the coin is fair. Simulation can show you how many ways the number of heads can occur and similarly the many ways assets and capital markets can behave. There are many alternative likely optimization inputs, efficient frontiers, and optimized portfolios consistent with your information. These alternatives are critically important for defining investment meaningful optimized portfolios.

But resampled efficiency is not simply a way of creating consistent alternative efficient frontiers and optimized portfolios. Resampled efficiency is an averaging process that distills all the alternative efficient frontiers into a new efficient frontier and set of optimized portfolios. Intuitively, resampled efficient portfolios are optimal with respect to the many ways assets and capital markets can perform in the investment period consistent with your forecasts.

³ J.D. Jobson and Bob Korkie, 1981. "Putting Markowitz Theory to Work." *Journal of Portfolio Management* 7(4): 70-74.

⁴ R. Michaud, 1989. "The Markowitz Optimization Enigma: Are Optimized Portfolios Optimal?" *Financial Analysts Journal*.

⁵ Richard Michaud, 1998. *Efficient Asset Management*. Oxford University Press, 1991. Originally published by Harvard Business School Press.

⁶ U.S. Patent # 6,003,018, December 1999.

The historical risk-return data and optimized portfolio weights for the six indicated asset classes in the table below provides a simple comparison of classical and resampled optimized near the middle of the frontier representing moderate risk portfolios. Note that, in this case, the classical portfolio has no allocation to small cap stocks. This is because the risk-return estimates are used literally and small caps have inferior estimates of return relative to large caps. In contrast, resampled efficiency includes a prominent component of small cap stocks. This is because resampled efficiency is sensitive to uncertainty and uses investment information in a more robust manner. While the two portfolios are comparable, the resampled efficient portfolio is better diversified and intuitively less risky. This result is general.

Classical vs. Resampled Optimal Portfolios⁷

Assets	Exp. Retn (%)	Std. Dev (%)	Classical (%)	Resampled (%)
T-bills	4.6	0.4	0	6
Interm. Govt.	7.1	4.3	36	18
LT Corp	8.3	6.4	0	12
LC Stock	17.7	13.4	64	54
SC Stock	15.7	17.4	0	10
Intern Stock	8.3	17.1	0	0

Resampled efficiency is very simply a better way to use investment information. It also has many investment attractive properties. Because resampled efficiency is an averaging process, it is very stable. Small changes in the inputs are generally associated with only small changes in the optimized portfolios. Resampled optimal portfolios are typically very investment intuitive without the need for constraints. This is because the uncertainty in your information, which is ignored with classical optimization, is being considered when defining the resampled optimized portfolio. Intuitive marketable optimized portfolios have the additional benefit of being a productivity tool, since few if any constraints are required to find suitable useful portfolios. Most importantly, resampled efficiency can be shown to improve investment performance, on average.

Resampled Efficiency is a new more practical definition of portfolio optimality. It is also a rigorous new patented tool for portfolio monitoring and rebalancing. This additional aspect of resampled efficiency is described further below.

Each portfolio on the resampled efficient frontier is the result of averaging a number of statistically equivalent efficient portfolios. This creates what may be described as a fuzzy set of statistically equivalent portfolios associated with each portfolio on the resampled efficient frontier. For a given resampled efficient frontier, the fuzzy region of associated portfolios, in its simplest form, can be visualized as a set of points filling the inside of an American football with the resampled efficient portfolio in the center.

⁷ The first two columns in the table display the annualized means and standard deviations of monthly data for six asset classes for 10 years. The last two columns display the classical and resampled optimal portfolio weights with equal annualized standard deviations of 9% in the middle of their respective frontiers.

Now think about comparing your portfolio to a resampled efficient frontier portfolio. Is your portfolio inside the football and close to the center or far from the center or even outside the football? If it's far from the center it's not statistically similar and you may want to consider rebalancing it. On the other hand, if your portfolio is close to the center, you may not need to rebalance the portfolio at all because it's statistically similar.

The rule computes a need-to-trade probability. A probability of 95% indicates that trading appears advisable; alternatively a 20% probability may indicate no need to rebalance. Because of the nature of the portfolio rebalancing problem, intuition is typically an unreliable guide. A portfolio may look the same as another and perform very differently and conversely. In many cases, reliable decisions require rigorous statistical analysis.

The resampled trading rule is the first statistically rigorous portfolio rebalancing rule available to the investment community. Avoiding trading without benefit enhances investment performance independent of the optimization process. The trading rule is also a highly scalable and automatable portfolio monitoring procedure. For a given resampled efficient frontier, hundreds of portfolios can be monitored with common laptop technology in a very short period of time. It is also easy to customize the monitoring process to accommodate client trading frequency mandates, changes in the certainty of investment information, and investment strategy trading patterns.

As it turns out, investment professionals were well advised not to use commercial portfolio optimizers because they did not take forecast uncertainty into consideration. However resampled efficiency, which explicitly considers investment information uncertainty, is a new class of portfolio optimization technology that holds out the promise of nearly automatable, customizable and effective asset management as well as new asset management tools not previously available. Thoughtful asset managers and advisors may now wish to reconsider their aversion to portfolio optimization tools.