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Resampled Efficiency™ vs. Bayes: Implications for Asset Management

by

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Abstract

Good inputs, prepared with Bayesian statistics, are no better than bad inputs if the portfolio construction process misuses investment information.

A recent study by Markowitz and Usmen (2003) found that the investment performance of Resampled Efficiency optimized portfolios (Michaud 1998) is superior to that of Markowitz (1959) mean-variance (MV) optimized portfolios with sophisticated Bayesian estimates of risk and return.[1] These results address and resolve an important open question on the relative statistical power of Bayes versus Michaud resampling methods in portfolio optimization. Good inputs may be no better than bad if the portfolio construction process misuses investment information.

Background

Since Michaud (1998, Ch. 6), investors have known that Resampled Efficiency improves investment performance on average.[2] Resampled Efficiency also features investment intuitive and marketable optimized portfolios with many attractive business management characteristics.[3] Resampled Efficient Frontier™ optimization generalizes MV optimization by allowing the user to condition the optimized portfolio according to the investor's level of certainty in the estimates. Resampled Efficiency is always preferable to MV because investors are never 100% certain of their estimates. Resampled Efficiency optimized portfolios are less risky because they are optimal relative to the many ways assets and markets may perform in the investment period.

The Open Question

The open issue concerning Resampled Efficiency was not whether investment performance on average was improved but whether the enhancement was marginal or substantive. In particular, will improvements in risk and return estimation dominate the performance enhancements of Resampled Efficiency? This question is of no small interest. Many institutions and investors devote the bulk of their human and capital resources to enhancing risk and return estimation and pay little attention to the portfolio construction procedures they use to convert their information into investment portfolios. Such resource management is suboptimal.

Bayes and Resampling

Resampling methods are widely used in modern statistics. They use Monte Carlo simulation to compute many statistically similar alternatives to enhance the information in a data set for analysis and estimation. Resampling is generally not misleading and often very useful.

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[1] Resampled Efficiency optimization was co-invented by Richard Michaud and Robert Michaud, U.S. patent 6,003,018, worldwide patents pending. New Frontier Advisors, LLC (NFA) is exclusive worldwide licensee.

[2] Many investors are familiar with back tests of performance rather than simulation studies. But back tests are period dependent. A good model may perform poorly and a poor model may perform well in some time period. Back tests have no statistical reliability for demonstrating investment performance. In contrast, simulation studies, used also by Markowitz and Usmen (2003), are the methods of choice for rigorously proving the effectiveness of a statistical procedure.

[3] Introductory and more advanced discussion can be found at www.newfrontieradvisors.com.

Bayesian methods are also widely used in modern statistics. Bayesian methods are statistical techniques optimally designed for improving estimates of statistical parameters. They improve the information in a data set by imposing a prior or guess on parameter estimation. Forecasts are improved because more information is added to the estimation process. However, in practice, some Bayesian methods may lead to perverse estimates. If the prior has misleading information, the investor may be worse off than using only the information in the data.

There are many Bayesian procedures in modern statistics ranging in application and analytical sophistication. Popular investment management applications based on the Bayes conditional probability formula in statistical texts include Black and Litterman (1990) and Michaud (1998, Ch. 11).[4]

Markowitz-Usmen Tests

Markowitz and Usmen (2003) perform ten different simulation tests choosing three different optimal portfolios in each case from Markowitz-Usmen Bayesian estimation and Markowitz optimization versus Resampled Efficiency without Markowitz-Usmen estimation. The Bayesian procedure in Markowitz and Usmen (2003) is a very sophisticated use of Bayesian, Monte Carlo, and numerical analysis methods for optimization input estimation. The "diffuse" prior method in Markowitz and Usmen is carefully designed to avoid computing misleading information about the data set.

Markowitz and Usmen were surprised to learn that their refined Bayes estimation process did not improve MV optimization sufficiently to dominate Resampled Efficiency (even with significantly inferior inputs). On average, Resampled Efficiency won in each of the ten tests for each of the three tested portfolios. They computed an average improvement of 57 bps for Resampled Efficiency.[5]

The Markowitz-Usmen results may appear counter-intuitive. Bayes methods are known to be powerful statistical tools for improving estimates of statistical parameters. Institutional methods for improving risk and return estimation are ubiquitous and presumably valuable. Yet the evidence in Markowitz-Usmen is that optimization methodology may be more important.

These results can be rationalized as follows. Risk and return estimates in practice are never known with perfect certainty. MV optimization is insensitive to estimation error and always results in "corner" or "error maximized" portfolios that are unlikely to perform well in the future. A lower level of information with an optimizer that is sensitive to estimate uncertainty and does not misuse investment information is likely on average to be more investment effective than superior inputs with MV optimization.

[4] For an example see Carlin and Louis (1996), p. 22. NFA's asset allocation software includes generalizations and enhancements of these procedures.

[5] See Markowitz and Usmen (2003) for further details of their tests.

Implications for Asset Management

Because very reliable risk and return estimation in practice is rare, Markowitz-Usmen teaches that portfolio construction methodology may be the dominant factor in optimized portfolio performance. The value of portfolio optimization that is sensitive to the uncertainty in investment information is far greater than is widely understood or appreciated in the investment community. The priorities and resource allocations of many asset management and consulting firms may need to be seriously reconsidered. The workarounds necessary for controlling MV optimization are generally unnecessary with Resampled Efficiency, which adds another factor to increased productivity and effectiveness. But Bayesian estimation and other methods for improving the reliability of risk and return estimates are not mutually exclusive with Resampled Efficiency. In the future, best practice may require more sophisticated statistical estimation procedures and Resampled Efficiency.

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