NEW FRONTIER ADVISORS, LLC

# Are Good Inputs Enough? No.

by

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## Abstract

Investment institutions tend to focus the bulk of their human and capital resources on developing reliable forecasts of asset risks and return while ignoring the optimization technology they use to transform their information into investors' portfolios. These good inputs are no better than bad if the portfolios that represent the information to the investor have no investment value.

Investment institutions tend to focus the bulk of their human and capital resources on developing reliable forecasts of asset risks and return while ignoring the optimization technology they use to transform their information into investors' portfolios. We will show that this is often a serious error. Good inputs are no better than bad if the portfolios that represent the information to the investor have no investment value.

Similarly, academics and theoreticians often propose statistical techniques such as Bayesian estimation to address the limitations of mean-variance (MV) optimization in practice. While statistical methods may improve the reliability of investment forecasts, good inputs alone typically do not overcome the investment limitations of MV efficient optimization.

Contrary to many popular beliefs, even excellent risk and return inputs are insufficient for overcoming the limitations of MV efficiency as a practical tool for asset management. We provide evidence that New Frontier Advisors, LLC (NFA) resampled optimization technology is a necessary procedure for good optimized portfolio performance. To prove this we first develop a set of very good inputs, detail our methodology and test the results with and without resampled efficiency™ optimization. We also discuss some new insights that show why resampled efficiency works better.

**Background**

Markowitz (1959) MV optimized portfolios have potentially many attractive investment characteristics. Optimized portfolios may reduce risk without reducing expected return. MV optimization also enables tailoring portfolios to various risk and return preferences.

But MV optimization misuses investment information. Tests show that MV efficiency in the presence of estimation error has very poor out-of-sample investment performance characteristics.[1] MV optimized portfolios "error maximize" the optimization inputs leading to portfolios that are typically investment unintuitive and have little investment value.[2]

A generalization of MV efficiency, called resampled efficient frontier™ optimization, leads to superior investment performance on average relative to MV efficiency.[3] Resampled efficiency controls estimation error by allowing the investor to condition the MV optimization according to an assumed level of forecast certainty.[4]

**Motivation**

Many institutions ignore the optimization technology that turns their investment information into an investor's portfolio. Their unexamined assumption is that good risk and return inputs are all that matter in an investment process. No one argues that reliable

[1] Jobson and Korkie (1981, 1980), Michaud (1998, Ch. 4).
[2] Michaud (1989).
[3] The resampled efficient frontier described in Michaud (1998, Ch. 6) is a U.S. patented technology, #6,003,018, co-invented by Richard Michaud and Robert Michaud, December 1999. Worldwide patents pending. New Frontier Advisors, LLC (NFA) has worldwide licensing rights.
[4] Michaud and Michaud (2003a) showed that an investor is better off on average using resampled efficiency with a misestimated forecast certainty level relative to MV efficiency which assumes a 100% certainty level.

investment forecasts are unimportant but are they sufficient to overcome the limitations of MV optimizers? The answer is, usually, no.

To show this, suppose you have found good estimates of future risk and return. What do you expect to see in your investment performance? You should expect that the efficient portfolio you compute from your information is roughly, on average, what you observe in the investment period. We develop such a set of inputs and then compare how they are used in MV and resampled efficiency optimization.

**The Test**[5]

Exhibit 1 displays the results of our optimization simulation tests.[6] Begin with your estimates of future risk and return for a universe of assets. From this information compute the MV efficient frontier. This is the black (top) curve in the exhibit.

Investors know that there is always uncertainty in investment forecasts. A way to model your uncertainty about the true risks and returns is to use Monte Carlo simulation or resampling to compute simulated returns from the original forecasts (black curve). For example, a Monte Carlo simulation of 100 returns for an asset with an assumed 10% return and 20% standard deviation will result in a mean and standard deviation different from the 10% and 20% inputs. The Monte Carlo estimates are a way of quantifying the effect of your uncertainty relative to the true risks and returns. From the simulated returns compute optimization inputs and the MV efficient frontier. By repeating the process many times you can compute many simulated efficient frontiers all of them consistent with your original forecasts and level of uncertainty. The average of the simulated efficient frontiers is displayed in the in-sample efficient frontiers in Exhibit 1. The red dash curve represents the average of the simulated MV efficient frontiers. For each simulated MV efficient frontier you can compute an associated resampled efficient frontier. The blue dash curve represents the average of the resampled efficient frontiers.

The dash curves represent in-sample efficient frontiers. These are the efficient frontiers you see when you go to invest. But because this is a simulation, we can go back to the original data, the black curve inputs, and see how estimation error led to misestimation of the original efficient frontier data set. This is the basis of the out-of-sample test of the optimization process. So for any simulated efficient frontier portfolio compute its mean and standard deviation from the inputs in the black curve. And do that for all the MV and resampled efficient portfolios.

The solid red curve represents the average risk and return of MV optimized portfolios with estimation error out-of-sample. The solid blue curve represents the average risk and return of resampled efficient optimized portfolios with estimation error out-of-sample.

[5] Readers of our newsletters and Michaud (1998) are likely to be aware of the nature of the optimization simulation tests that are used to demonstrate how optimizers perform out-of-sample. We reprise the key components here but recommend reading Michaud (1998, Ch. 4) for further details.

[6] The exhibit and analysis are taken from Michaud and Michaud (2003b).

There is one additional step that has been ignored up until now. In this test the simulated optimization inputs are Stein adjusted.[7] After the returns are simulated for each efficient frontier the Stein procedures are applied to compute the optimization inputs. This procedure is the theoretically optimal way to shrink estimates to more sensible values and is similar to a Bayesian estimate of portfolio optimization inputs with a diffuse prior. In effect, these are very "good" inputs from a modern statistical point of view.[8]

**Results**

The dash and solid blue curves represent the in-sample and out-of-sample resampled efficient frontiers. The curves intersect. Given the congruity between forecasts and actual performance on average, we can conclude that the inputs are excellent.

The dash and solid red curves representing the in-sample and out-of-sample MV efficient frontiers are far apart. The in-sample MV efficient frontiers overestimate the return associated with portfolio optimization not only with respect to resampled efficiency (blue dash curve) but importantly with respect to out-of-sample investment performance (red solid curve). Even with excellent inputs, MV efficiency error maximizes the risk and return inputs, creates upward biased estimates of future performance, and substantially underperforms resampled efficiency! The same reliable investment information that performed so well with resampled efficiency is badly misused by MV efficiency. In addition, the error maximization property of MV efficiency means that real estimates such as trading costs are misused in the optimization process; you are likely to think that the returns are much higher relative to trading costs than they actually are.

**The Bottom Line**

These results should trouble many investment institutions. Resource allocation bias toward investment forecasting and away from effective optimization technology is self-defeating. While good inputs are important, they need to be transformed into optimized portfolios that do not misuse the information.

What we have demonstrated is that resampled efficiency is a necessary condition for investment effective portfolio optimization. The resampling process unbiases the optimization process so that the information is transferred directly into the optimized portfolios.

It should also be clear that resampled efficiency is not inconsistent with any input optimization process that may reduce estimation error. The better the input estimates in a resampled optimization the more likely investment performance is improved. In particular, Bayes and Stein estimation in conjunction with resampled efficiency holds out the promise of dramatically improved optimized portfolio performance in the 21st century.

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[7] The two Stein estimation procedures are James-Stein-Efron-Morris for return and Ledoit for covariance estimation. See Michaud (1998, Ch. 8) for further information and references.

[8] In these tests the Stein estimates of optimization inputs are surrogates for reliable forecasts of risk and return and are not meant to replace the process of developing reliable investment information from economic, market, and other sources.

**Understanding Resampled Efficiency**
Unlike MV efficiency, resampled efficiency is an out-of-sample definition of portfolio optimality. MV efficiency is correct only when there is no uncertainty in the optimization inputs. But investment forecasts are always uncertain. Resampled efficiency deals with your uncertainty by simulating all the many ways markets and assets can perform based on your forecasts and then finding portfolios that, on average, do well in all the simulated outcomes.

Portfolio optimization with uncertainty implies fundamental changes in investor perceptions. The MV efficient frontier familiar to students of finance and investment professionals turns out to be essentially useless in understanding portfolio optimality. In particular where a portfolio plots in the mean-variance diagram does not determine how it performs.

Exhibit 2 illustrates the usual in-sample relationship between the MV and resampled efficient frontiers. Now consider a portfolio that plots at point A above the resampled and below the MV frontiers. Is portfolio A more efficient than portfolios on the resampled efficient frontier? Do you prefer investment in portfolio A to a portfolio with similar risk on the resampled frontier?

By definition, portfolios on the resampled efficient frontier are optimal conditional on your level of forecast certainty. The portfolio that plots at A is not preferable. Intuitively, the asset weights are "too active" relative to the level of information certainty in your inputs. But there is an additional point that helps to further clarify the issue. A portfolio that plots at point A is not unique. There are an infinite number of portfolios with the same mean and variance as A. There may even be one-asset portfolios that plot at A yet have much risk out-of-sample and unlikely to be efficient by anyone's definition.

The underlying financial reality explained by resampled efficiency is that it is the structure of the portfolio, not its mean and variance parameters, that defines optimality and how a portfolio performs out-of-sample.[9] This is what has been missing in our understanding of portfolio efficiency for nearly fifty years.

**Conclusion**
Portfolio structure relative to out-of-sample performance conditional on forecast certainty characterizes a more useful definition of portfolio optimality in investment practice. Portfolio optimality defined by out-of-sample investment performance reveals many investment illusions that currently negatively affect much investment practice.[10] Resampled efficiency offers important new investment tools and more effective and intuitive asset management.

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[9] For example, rebalancing or other procedures based on a portfolio's mean and variance parameters are unlikely to have useful investment value out-of-sample.
[10] We will be reporting on some of these in future newsletters.

Exhibit 1
Michaud (1998) Data
In- and Out-of-sample tests, JS Estimated

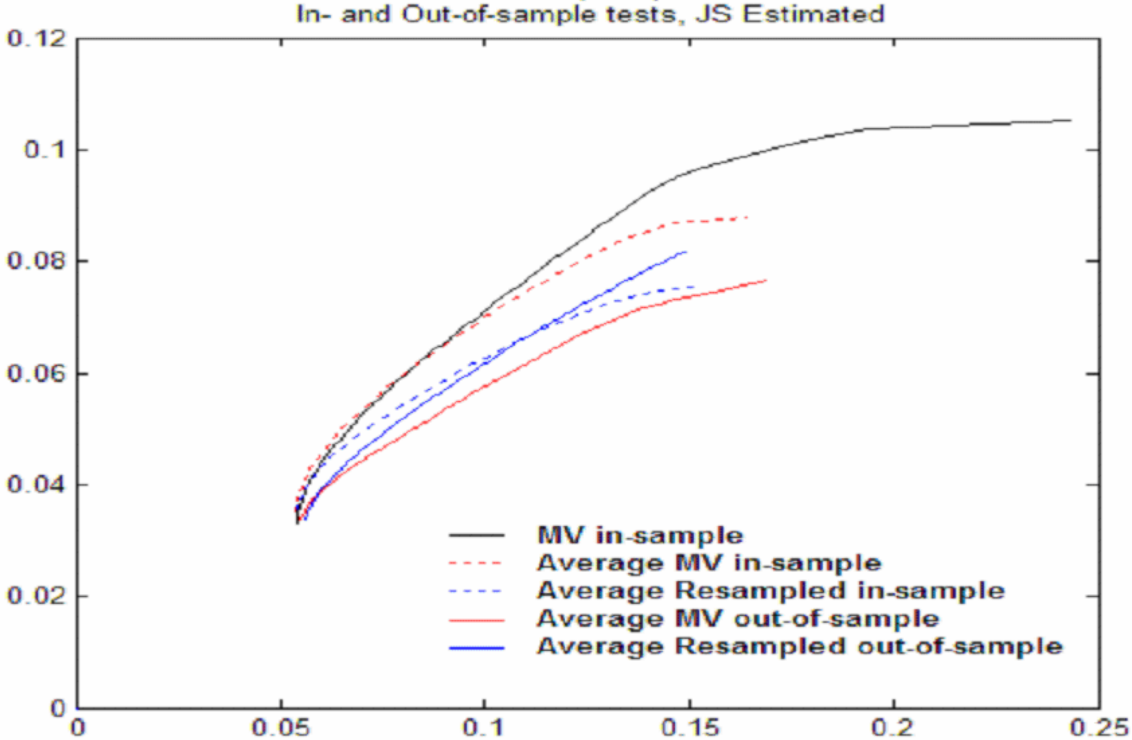
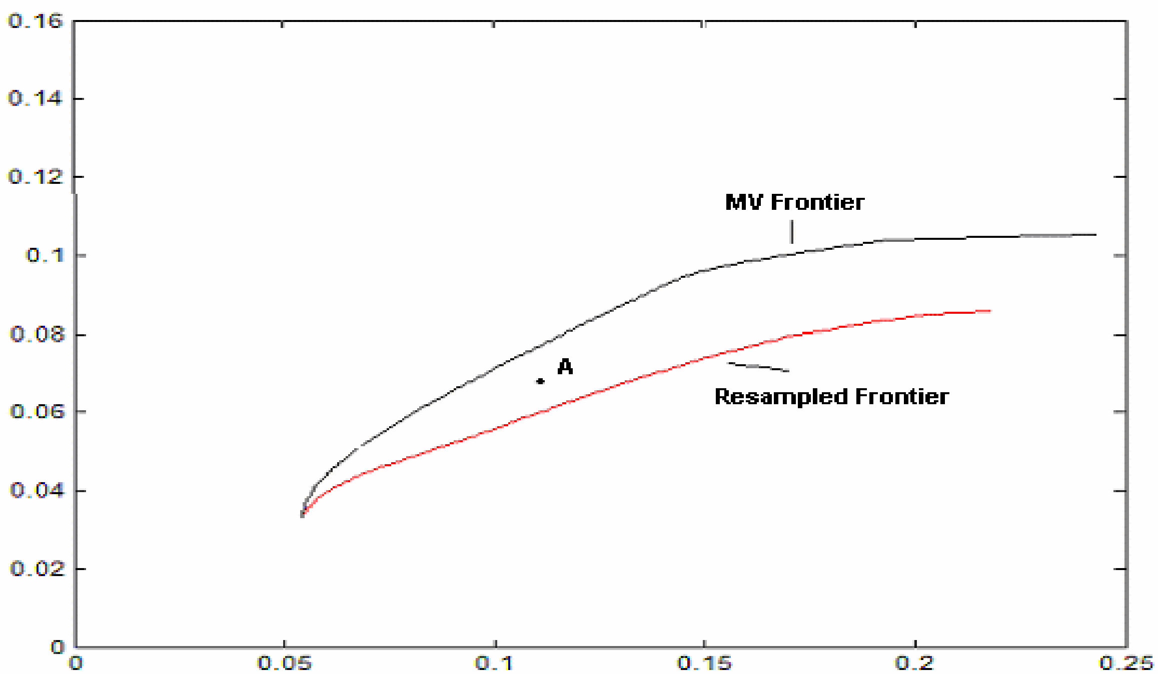



Exhibit 2
MV and Resampled Efficient Frontiers



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