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# The "Fundamental Law of Active Management" Is No Law of Anything[1]

By
Richard O. Michaud, David N. Esch, and Robert O. Michaud[2]

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[2] Richard O. Michaud is CEO of New Frontier Advisors, LLC, Boston, MA 02110; David N. Esch is Managing Director of Research of New Frontier Advisors LLC.; Robert O. Michaud is CIO of New Frontier Advisors, LLC.

## Abstract

Roughly half of all professionally managed funds globally employ optimized portfolio design principles that are applications of Grinold's "Fundamental Law of Active Management." These include: Invest in many securities, use many factors to forecast return, trade frequently, and optimize with minimal constraints. We show with simple examples followed by rigorous simulation proofs that these proposals are invalid and self-defeating. This is because estimation error and required economically valid constraints are ignored in derivations. These flawed principles have been unchallenged by academics and practitioners for nearly twenty-five years and may adversely impact a trillion dollars or more in current fund management.

Benchmarks arise naturally in judging asset manager competence and for meeting investment goals. An active investment manager typically claims to provide enhanced return on average relative to a given benchmark or index for a given level of residual risk. The information ratio (IR) – estimated return relative to benchmark per unit of residual risk or tracking error – is a convenient and ubiquitous framework for measuring the value of active investment strategies.

The Grinold (1989) "Fundamental Law of Active Management" asserts that the maximum attainable IR is approximately the product of the Information Coefficient (IC) times the square root of the breadth (BR) of the strategy.[3] The IC represents the manager's estimated correlation of forecast with ex post residual return while the BR represents the number of independent bets or factors associated with the strategy. Grinold and Kahn (1995, 1999) assert that the "law" provides a simple framework for enhancing active investment strategies. While a manager may have a relatively small amount information or IC for a given strategy, performance can be enhanced by increasing BR or the number of bets in the strategy. In particular they state: "The message is clear: It takes only a modest amount of skill to win as long as that skill is deployed frequently and across a large number of stocks."[4] Their recommendations include increasing trading frequency, size of the optimization universe, and factors to models for forecasting return. Assumptions include: independent sources of information and IC the same for each added bet or increase in BR.

Clarke, de Silva and Thorley (2002, 2006) (CST) generalize the Grinold formula by introducing the "transfer coefficient" (TC). TC is a scaling factor that measures how information in individual securities is "transferred" into optimized portfolios. TC represents a measure of the reduction in investment value from optimization constraints. This widely influential article has been used to promote many variations of hedge fund, long-short, alternative, and unconstrained investment strategies.[5]

A significant literature exists on applying the Grinold law and variations for rationalizing various active equity management strategies. Extensions include: Buckle (2004), Qian and Hua (2004), Zhou (2008), Gorman et al (2010), Ding (2010), Huiz and Derwall (2011). Industry tutorials and perspectives include Kahn (1997), Kroll et al (2005), Utermann (2013), Darnell and Ferguson (2014). Teachings include the Chartered Financial Analyst (CFA) Institute Level 2, the Chartered Alternative Investment Analyst (CAIA) Level 1 and many conferences and academic courses in finance. Texts discussing the formula and applications include Focardi and Fabozzi (2004), Jacobs and Levy (2008), Diderich (2009), Anson et al (2012), Schulmerich et al (2015). Roughly half of globally professionally managed funds are estimated to employ optimized portfolio design principles that are applications of Grinold's "Fundamental Law."

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[3] The Grinold formula is analytically derived and based on an inequality unconstrained maximization of a quadratic utility. It should not be confused with Markowitz (1952, 1959) which assumes linear (inequality and equality) constrained portfolios and requires quadratic programming techniques to compute the MV efficient frontier. In particular, the Markowitz efficient frontier is generally a concave curve in a total or residual return framework while in Grinold (see e.g., GK 1995, p. 94) it is a straight line emanating from a zero residual risk and return benchmark portfolio. The Grinold derivation also assumes IC to be small, in the order of 0.1.

[4] GK (1995, Ch. 6, p. 130), also GK (1999, Ch. 6, p. 162).

[5] One example is Kroll et al (2005). Michaud (1993) was the first to note possible limitations of the long-short active equity optimization framework.

We show with simple examples followed by rigorous simulation proofs that these proposals are invalid and self-defeating. This is because estimation error and required economically valid constraints are ignored in derivations. Naively seeking to add breath may increase the complexity of the problem to the extent that it reduces rather than improves IR, and the outcome will fall far short of expectations. The "law" is not a law of anything and does not help you invest better. These flawed principles have been unchallenged by academics and practitioners for nearly twenty-five years and possibly adversely impact as much as a trillion dollars or more of managed assets in current investment practice.

The outline of the paper is as follows. Section 1 presents the Grinold formula, the GK and CST prescriptions for active management and a review of the GK casino management rationale. Section 2 discusses the limitations of the GK and CST prescriptions from an intuitive investment perspective. Section 3 provides a discussion of simulation tests, properties of index-relative mean-variance (MV) optimization and three earlier simulation studies relevant to our results. Section 4 presents our Monte Carlo simulation studies that confirm that the principles associated with the fundamental law are invalid and likely self-defeating. Section 5 provides some practical recommendations for a well-defined portfolio optimization and investment management program. Section 6 provides a summary and conclusions.

## 1.0 Grinold's Fundamental Law of Active Management

The Grinold (1989) formula is an approximate decomposition of the information ratio (IR) generally associated with active investment management. Grinold shows that the MV optimization of an inequality unconstrained residual return investment strategy is approximately proportional to the product of the square root of the breadth (BR) and the information correlation (IC).[6] Mathematically,

$$IR \cong IC * \sqrt{BR}$$

where IR = information ratio = (alpha) / (residual or active risk)
IC = information correlation (ex ante, ex post return correlation)
BR = breadth or number of independent sources of information.

The uncontroversial wisdom of the formula is that successful active management depends on both the information level of the forecasts and the breadth associated with the optimization strategy. However, Grinold and Kahn (GK) (1995, 1999) and Clarke, deSliva, and Thorley (CST) (2002, 2006) go further. They apply the Grinold formula to assert that only a modest amount of information (IC) is necessary to win the investment game simply by sufficiently increasing the number of assets in the optimization universe, the number of factors in a multiple valuation framework, more frequent trading and reducing optimization constraints.

There are two fundamental reasons for limitations of the principles associated with the Grinold law for practical asset management: 1) the formula ignores the impact of estimation error in investment information on out-of-sample optimized investment performance; 2) the formula assumes a quadratic utility unconstrained optimization framework that ignores the necessity of including economically meaningful inequality constraints required for defining portfolio

[6] The detailed derivation is given in GK, Ch. 6, and Technical Appendix.

optimality in practice. We will demonstrate that the GK and CST prescriptions associated with the formula are invalid and self-defeating and not recommendable for practice.

## 1.1 The Casino Game Rationale

GK use a casino roulette game to rationalize applications of the Grinold formula to asset management in practice.[7] The probability or IC of a winning play (for the casino) of the roulette game is small but more plays (breadth) lead to the likelihood of more wealth. However, there are important differences between the play of a roulette game in a casino and the play of an investment game in practice. In the casino context, the IC is stable, known, positive, and constant across plays. In an investment game the IC is unstable, has estimation error, and may be insignificantly or even negatively related to return. Plays of the investment game may not be independent and increasing the number may be undesirable. While interesting the casino game rationale for rationalizing applications of the Grinold formula to investment practice is invalid.

## 2.0 Discussion of GK and CST Prescriptions

GK and CST propose four principles of optimized portfolio design for enhanced investment value in an index-relative MV optimization framework. We discuss the limitations of each prescription in turn from an intuitive point of view.

## 2.1 Large Optimization Universe Fallacy

GK argue that investment value increases with the size of the optimization universe conditional that the IC is roughly equal for all securities in the optimization universe. How realistic is this assumption?

For a small universe of securities the assumption of uniform average IC may be tenable. Small universes may be fairly homogeneous in character. However, for a large and expanding optimization universe, it seems untenable to assume uniform average IC across all subsets. Any manager will naturally use the securities with the best information first. While, theoretically, adding more assets may add marginally to breadth, all other things the same, it is also likely to result in less predictable securities and reduce the overall average IC level of the universe. A lower average IC may cancel any gains made from increasing breadth.

The issue can be framed in a more common practical setting. Consider an analyst suddenly asked to cover twice as many stocks. Given limitations of time and resources, it is highly unlikely that the analyst's average IC is the same for the expanded set of stocks. Issues of resources and time rationalize why analysts tend to specialize in areas of the market or managers in investment strategies that limit the number of securities that they cover. In practice many traditional managers limit the number of securities they include in their active portfolio to not much more than twenty or fifty. Except for relatively small asset universes, the average IC and overall level of IR may often be a decreasing function of the number of stocks in the optimization universe, all other things the same. Grinold and Kahn seem to be aware of these limitations, for example as suggested by their statement "The fundamental law says that more breadth is better, provided

[7] The casino roulette game framework is very consistent with the assumptions used in the Grinold derivation in GK (1995, 1999, Ch. 6. App.)

the skill can be maintained." Nevertheless, average IC and optimization universe size are inevitably related especially for large universes of assets.

## 2.2 Multiple Factor Model Fallacy

Large stock universe optimizations typically use indices such as the S&P500, Russell 1000 or even a global stock index as benchmarks. In this case individual analysis of each stock is generally infeasible and analysts typically rely on factor valuation frameworks for forecasting alpha. For example, stock rankings or valuations may be based in part on an earnings yield factor.[8] As GK note, if earnings yield is the only factor for ranking stocks, there is only one independent source of information and the breadth equals one.

In the Grinold formula, the IR increases with the number of independent positive significant factors in the multiple valuation forecast model. However, in practice, asset valuation factors are often highly correlated and may often be statistically insignificant providing dubious out-of-sample forecast value.[9] Finding factors that are reasonably uncorrelated and significantly positive relative to ex post return is no simple task.

Factors are often chosen from a small number of categories considered to be relatively uncorrelated and positively related to return such as value, momentum, quality, dividends, and discounted cash flow.[10] In experience, breadth of multiple valuation models is typically very limited and unlikely to be very much greater than five independent of the size of the optimization universe.[11,12] As in adding stocks to an optimization universe, adding factors at some point is likely to include increasingly unreliable factors that are likely to reduce, not increase, the average IC of an investment strategy.

Michaud (1990) provides a simple illustration of adding factors to a multiple valuation model. While adding investment significant factors related to return can be additive to IC, it can also be detrimental in practice. There is no free lunch. Adding factors can as easily reduce as well as enhance forecast value, and the number of factors that can be added while maintaining a desirable total IC is severely limited in practice.

## 2.3 Invest Often Fallacy

GK recommend increasing trading period frequency or "plays" of the investment game to increase the BR, and thus the IR of a MV optimized portfolio. The Grinold formula assumes trading decision period independence and constant IC level. However, almost all investment

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[8] Some standard methods for converting rankings to a ratio scale to input to a portfolio optimizer include Farrell (1983) and references. Michaud (1998, Ch. 12) notes some common scaling errors.

[9] There is a limit to the number of independent investment significant factors even in many commercial risk models, often far less than ten.

[10] Standard methods such as principal component analysis for finding orthogonal risk factors are seldom also reliably related to return.

[11] See e.g., Michaud (1999).

[12] While principal component or factor analysis procedures for identifying orthogonal factors in a data set may be used, most studies find no more than five to ten investment significant identifiable factors that are also useful for investment practice.

strategies have natural limits on trading frequency.[13] For example, an asset manager trading on book or earnings to price will have significant limitations increasing trading frequency smaller than a month or quarter. Reducing the trading period below some limit will generally reduce effectiveness while increasing trading costs.

Fundamentally, trading frequency is limited by constraints on the investment process relative to investment style.[14] Deep value managers may often be reluctant to trade much more than once a year while growth stock managers may want to trade multiple times in a given year. Increased trading, to be effective, requires increasing the independence of the trading decision while maintaining the same level of skill. This will generally require increased resources, if feasible, all other things the same. The normal trading decision period should be sufficiently frequent, but not more so, in order to extract relatively independent reliable information for a given investment strategy and market conditions.

It is worth noting that the notion of normal trading period for an investment strategy does not imply strict calendar trading. Portfolio drift and market volatility relative to new optimal may require trading earlier or later than an investment strategy "normal" period. In addition a manager may need to consider trading whenever new information is available or client objectives have changed. Portfolio monitoring relative to a normal trading period including estimation error is further discussed in Michaud et al (2012).

## 2.4 Remove Constraints Fallacy

Markowitz's (1952 1959) MV optimization can accommodate linear equality and inequality constraints. In actual investment practice, MV optimized portfolios typically include many linear constraints. This is because unconstrained MV optimized portfolios are often investment unintuitive and impractical. Constraints are often imposed to manage instability, ambiguity, poor diversification characteristics, and limit poor out-of-sample performance.[15] However, constraints added solely for marketing or cosmetic purposes may result in little, if any, investment value and may obstruct the deployment of useful information in risk-return estimates.

In general, inequality constraints are necessary in practice. Inequality constraints reflect the financial fact that even the largest financial institutions have economic shorting and leveraging limitations. Recently, Markowitz (2005) demonstrates the importance of practical linear inequality constraints in defining portfolio optimality for theoretical finance and the validity of many tools of practical investment management. Long-only constraints limit liability risk, a largely unmeasured factor in most portfolio risk models and often an institutional requirement. Regulatory considerations may often mandate the use of no-shorting inequality constraints. Performance benchmarks may often mandate index related sets of constraints for controlling and monitoring investment objectives. Moreover, inequality constraints limit the often negative impact of estimation error in out-of-sample performance (Frost and Savarino 1988).

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[13] Special cases may include proprietary trading desk strategies where the information level is maintained at a reasonable level and trading costs are nearly non-existent. Other cases, such as high frequency and algorithmic trading are arguably not investment strategies but very low level IC trading pattern recognition relative to highly sophisticated automated liquidity exchange intermediation.

[14] Trading costs and market volatility are additional considerations.

[15] Michaud (1989).

## 3.0 Testing GK and CST proposals

Investment managers often use a back test to demonstrate the likely value of a proposed investment strategy. In this procedure a factor or strategy is evaluated on how it performed for some historical data over some time period. While a back test may often be the only practical framework for testing an investment hypothesis, no reliable prospective information is possible by definition. Moreover, back tests are notorious for misleading investors, resulting in loss of wealth, careers, and dissolution of firms. Investors should be keenly aware of the serious limitations of any back test as evidence of the reliability of any factor relationship or investment strategy.[16]

A simulation study can be a far more reliable alternative framework for testing the value of optimized investment strategies. Such a procedure evaluates the likely out-of-sample performance of an in-sample optimized portfolio under many realistic investment scenarios. Performed properly it can provide definitive proof of the out-of-sample value of a proposal for adding investment value.

In the following sections we explain the summary statistics used to evaluate the out-of-sample performance of investments following the prescriptions of fundamental law, describe the simulation test framework in greater detail, and discuss the results of three important simulation experiments that support our results.

## 3.1 Properties of index-relative MV optimization

Index-relative MV optimization is total return MV optimization with an index weight sum to zero constraint. The index has no risk and no return by definition. In the unconstrained case the IR efficient frontier is a straight line starting from the origin with slope equal to IR. In the sign constrained case the IR efficient frontier is a straight line rising from the origin with slope equal to max IR until the first pivot portfolio on the total return MV efficient frontier.

Roll (1992) provides the classic critique of the IR active return MV optimization framework. He shows that IR optimized portfolios are dominated by portfolios in MV total return space unless the index is assumed total return MV efficient. Merton (1987) provides a rational market framework for index-relative MV optimization. Under relatively straightforward conditions consistent with many active asset management strategies common benchmarks may often be considered total risk and return MV efficient. Presuming economic rational agents, we assume the index chosen is total risk and return MV efficient. This assumption is a best case scenario for the investment value of an active investment strategy.

Assuming Merton MV efficiency, all index-relative MV optimized portfolios are equivalently MV total return efficient. There are an infinite number of possible IR efficient portfolios to represent the investment value of an index-relative optimized investment strategy. This is because max IR efficiency is ambiguous. This is consistent with index-relative optimization practice where the IR maximized portfolio is defined relative to a specified level of tracking error to the benchmark. In order to compare the out-of-sample characteristics of an optimized investment strategy it is

[16] Even long-term academic studies remain susceptible to the unreliability of results of any back test.

convenient to focus on one relevant optimized portfolio. Note that the MSR portfolio is IR efficient on both the unconstrained and sign constrained efficient frontier under our assumptions. It is also a convenient single portfolio to represent portfolio optimality relative to the MV inputs. In the following the in-sample optimal MSR portfolio is used to represent the out-of-sample investment value of max IR investment strategies for both unconstrained and sign constrained investment strategies.

### 3.2 Portfolio simulation study framework

In a typical simulation study, a referee is assumed to know the true means, standard deviations, and correlations for a set of assets and consequently the true MSR for portfolios of those assets. The players, each representing an investment strategy, do not know the referee's true parameters. Instead, they receive simulated returns from the referee, so they can only observe the truth obscured by estimation error, as is true for all real-world investment managers. The players then compute optimal weights for their strategies and send them back to the referee to score. The referee determines the winner for that simulation. The procedure is repeated many times for a range of referee scenarios, and averages and various statistics computed for each player. In this way the out-of-sample performance of each strategy can be compared, and the better overall strategy determined.

### 3.3 Three important MV optimization simulation studies

Jobson and Korkie (1981) (JK) provide the classic study of the effect of estimation error on the out-of-sample investment value of inequality unconstrained MV optimized portfolios. Note that the JK study applies equivalently to inequality unconstrained quadratic utility portfolio optimization, a framework widely used in financial theory and development of many investment strategies. In their study the referee's truth is based on historical MV inputs for twenty stocks. They compute simulated MV inputs reflecting five years of monthly return data. They find that the average of the true SRs of the simulated MSR optimal portfolios was twenty-five percent of the true MSR optimal portfolio. In addition they show that equal weighting substantially outperforms the optimized portfolios.[17] They conclude that unconstrained MV optimization is not recommendable.

Frost and Savarino (1988) (FS) perform a similar simulation study that compares inequality unconstrained to long-only MV optimized portfolios. They find that long-only MV optimized portfolios dominate inequality unconstrained. This is because sign constraints often limit the impact of estimation error improving out-of-sample investment performance. This early result directly contradicts the conclusions in CST. Economically valid constraints often act like Bayesian priors focused on portfolio structure enforcing rules representing legitimate information not contained in the optimization inputs. Such restriction can mitigate estimation error in risk-return estimates implicitly by forcing the solutions towards more likely optimal portfolios.

Markowitz and Usmen (2003) demonstrate the superiority of the Michaud resampling methodology (Michaud and Michaud, 1998, 2008a, 2008b), over classic MV optimization, even

[17] An equal weighted portfolio is a simple way to compare the optimality of unconstrained optimized portfolios. Note that equal weighting includes inequality constraints by definition. Inequality constraints, an integral part of the Markowitz (1952, 1959) framework, are often found to be important in defining investment superior optimized portfolios.

when the MV player uses a superior input estimation methodology. This study uses the simulation study framework described in section 3.2, and provides definitive evidence that resampling increases investment value in an asset allocation with a realistic incomplete information setting.[18]

These three studies all demonstrate a robust effect for their research hypotheses, across a wide range of investment scenarios, as opposed to back tests, which demonstrate performance only within one particular market scenario. Our simulation experiment, presented in detail in the next sections, confirms all of these important results.

## 4.0 GK and CST Simulation Proofs

In this section we develop rigorous simulation studies to address the limitations of the GK and CST prescriptions for practice.

## 4.1 Simulating Breadth while Maintaining Information Levels

The Grinold formula posits a square root relationship between breadth (BR) and IR for a given level of skill (IC). Increasing the size of an optimization universe is often assumed to be a simple way of adding breadth (BR) to an investment strategy and improving likely out-of-sample investment performance all other things the same, provided that specific information is available for the added securities. To test the effect of adding breadth to investment performance, we need to simulate added breadth while maintaining a designated IC.

Although the number of assets is not conceptually identical to breadth as specified by GK and CST, our particular construction of the simulation experiment guarantees that we are adding one unit of breadth for each asset added to the case. This is true because the signal component of the IC, specific for the new asset, is added independently to the system, and corresponding asset-specific information is also being added to the covariance simulation. We are still adding breadth up to the last increment in portfolio size, and the diminishing added performance relative to the Fundamental Law's prediction cannot be explained as failure to add breadth when increasing universe size.

## 4.2 Simulation Methodology

We calculate a truth for the purposes of simulation based on recent monthly data taken from major US Stock exchanges.[19] Using estimates derived from real market data for our simulation parameters provides good coverage of the return distribution from a recent history (1994-2013) of US market data, while our methodology guarantees a limitless supply of simulated breadth at a constant IC.

We use the direct estimates from the real twenty year histories to provide a realistic range and probability distribution for our simulated return draws. From this so-called "uber-truth," we simulate a range of "truths," which are used to simulate the observed returns and in particular MV

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[18] Campbell et al (2008) dispute the results in Markowitz and Usmen (2003). However Michaud and Michaud (2008c) note critical limitations in the Campbell et al (2008) study.

[19] We include all listed U. S. stocks in the CRSP database that had twenty years of continuous monthly returns from January 1994 to December 2013. We excluded returns greater than 50% or less than -50%. We found 1681 stocks that met our criteria. Parallel experiments with shorter histories were also run to investigate if selection bias affects results, with no positive findings, so we present the twenty year history here.

inputs given to each of the optimizer treatments. Each truth is meant to be an alternative but still plausible market scenario. Our particular simulation process guarantees a particular average IC[20] for the estimation of expected return.

A particular IC implies a particular signal-to-noise ratio in the alpha estimation process. All estimation processes lie on a spectrum from no information (IC=0) to complete information (IC=1). Hence, there is a certain level of information in the covariance estimation process, which may or may not correspond exactly to the information in the alpha estimation, resulting in an imperfect estimate of the covariance. However, since IC as a measure of noise is not well suited to characterizing estimation of variance parameters, it is not clear how to persuasively match the noise level implicit in an IC specification to simulation of a covariance matrix in a simulation experiment. Moreover, in practice, the covariance estimation often comes from a completely different source of information, such as a commercial factor model, so it may naturally have a different information content from the alpha estimation process. Best practices for estimating covariance matrices are widely debated in the finance community, with certain estimators preferred over others depending on how the estimate will be used. We do not wish to enter that debate in this article, but more on the topic can be found in Fan Fan and Lv (2008), who recommend a factor-based estimate of covariance for its stability under inversion, and hence optimization. Since our simulation process did not naturally extend to include, for example, Fama-French (1992) factors, without some controversial distributional assumptions, we did not follow their recommendations to generate covariance estimates in our study. In order to leave no doubt about the possible negative impact of covariance estimation on performance of the optimizations, we used two different treatments for covariance inputs in our experiment. Firstly, to demonstrate a realistic situation with some controlled estimation error, we used a Ledoit (2003) estimator from data. This estimate has properties which limit the damage done by inversion or optimization, but nevertheless has some impact on performance as the cardinality of the optimization increases. Secondly, in order to completely eliminate suspicion that the covariance may be causing the observed failure of the fundamental law, we used a perfectly estimated error-free (referee's) covariance matrix within some of the simulated optimizations. In this case, the observed performance shortfall then must be attributed to the error implied by the noisy alpha estimate, calibrated to a particular IC, and therefore the failure of the fundamental law itself. The perfect covariance provides the definitive proof that the fundamental law is not an accurate description of investment value as a function of IC and BR . For all subsets of the large master dataset, we match securities for forming the basis of each truth, i.e. expected return and covariance parameters are simulated from the same securities, in order to maintain the natural relationship between means and variances that exists within the data source.

IC alone is insufficient to determine the suitability of a return estimate for optimization. Any corresponding estimate with a particular IC for a set of securities can be scaled positively or added to any constant without altering IC. Thus wildly unsuitable and badly optimizing return estimate sets can be created with the same IC as an estimate that optimizes and performs well. Beyond IC, location (mean) and scale (standard deviation) of a set of return estimates also impact the investment value of an optimized portfolio. In order to demonstrate a consistent and realistic level of performance for a particular level of IC, we need to make sure the return estimates are appropriately scaled and have a reasonable global mean, based on information that would

[20] Details of simulating with a particular IC are given in Esch (2015).

typically be available to an analyst. Setting the mean and standard deviation of the vector of return estimates equal to those of a short (e.g. 5-year) simulated history is a reasonable way to ensure good performance. Portfolios optimized from inputs scaled this way have Sharpe ratios nearly equal to those attained from "cheating" and using information derived from the truth in the simulated optimizations for scaling the simulated inputs.

Thus, with complete mean and variance inputs we proceed to create optimized portfolios via three methods: unconstrained maximum Sharpe ratio, maximum Sharpe ratio with positivity constraints, and equal weighting. Of course numerous other methods are possible, but not presented here due to space limitations. Maximum Sharpe ratios are then calculated for each method using the truths, which were unknown to the optimizer treatments. These are different from the in-sample Sharpe ratios that an investor would calculate using his or her own estimates and would not be available in real-world practice. Generally the in-sample estimates, created from the manager's inputs, are far too optimistic, and indeed, although they are not shown on these graphs, their ranges dominate the others on the graph.

In the simulation studies that follow we differentiate two cases that reflect investment practice: asset allocation and equity portfolio optimization strategies. Asset allocation strategies typically include five to thirty securities and rarely more than fifty. On the other hand equity portfolio optimization strategies may include hundreds or even thousands of assets in the optimization universe. While active equity asset managers may often claim to have an IC level of approximately 0.10, an important assumption in the derivation of the Grinold formula, it may be useful to bracket our results with a more optimistic IC of 0.25, illustrated in the small universe asset allocation case.

### 4.3 Simulation Results

Figures 1 and 2 each consist of two panels of simulation results for various IC levels and two different sizes of optimization universes. Figures 1A and 1B display average Sharpe ratios for universe sizes up to 50 stocks representing the asset allocation case for IC levels of either 0.10 or 0.25. Figures 2A and 2B display ratios for universe sizes up to 500 stocks representing the equity portfolio optimization case for IC levels of 0.10 for either of two different estimation methods for the covariance. Each value presented on the graphs is averaged from 10,000 optimizations resulting from 1000 truths simulated from the selected universe. The three graphed series in each panel show progressions of average Sharpe ratios resulting from three different optimization methods. The "unconstrained" series displays the out-of-sample averages of the simulated unconstrained MSR portfolios, the "equal weight" series displays the average Sharpe ratios of equal weighted portfolios, and the "constrained" series reflects the average Sharpe ratios of out-of-sample simulated long-only MSR portfolios.

The results of our experiments for the asset allocation cases from Figure 1 demonstrate a definite failure of the GK and CST specifications of the fundamental law of management. Specifically, the improvement is less than it should be with regard to both breadth and IC. Increasing the breadth does improve the average Sharpe ratio, but by diminishing margins and by less than the square root of breadth. Increasing the IC from 0.10 to 0.25 produces surprisingly little improvement in Sharpe ratio performance out-of-sample. In Figure 1A, the unconstrained portfolios dramatically underperform both sign constrained and equal weighting. While adding assets increases the Sharpe ratios of unconstrained portfolios out-of-sample, the gain is minimal and, we will argue below, unrealistic. How positivity constraints help the optimization process depends on the quality of information and universe size but the results generally contradict the CST view that eliminating constraints adds investment value.[21] The naïve analyst may think *a priori* that performance will increase because of increased IR forecasts calculated with estimates used in the optimization, but such in-sample calculation amounts to assuming perfect information and estimation ability, clearly unrealistic for investors of any skill level. Our results vividly demonstrate the hazards of ignoring estimation error when optimizing.

Figure 2 presents similar simulation experiments to Figure 1 for expanded optimization universes of up to 500 securities. One clear difference in the large universe case is the overall inferiority of equal weighting particularly given the presence of significant levels of information. A second important difference is that the benefit of positivity constraints depends crucially on the level of presumed forecast information. For a typical level of IC = 0.10, sign constrained large universe optimization provides enhanced performance relative to unconstrained for much of the size spectrum. Figure 2B shows the results using the referee's covariances as optimization inputs. We see that the impact of no estimation error in the covariance has a positive effect on the out-of-sample performance of the optimizations. For larger universe sizes, the out-of-sample penalty in Sharpe ratio performance for covariance estimation under positivity constraints is greater than for the unconstrained case, for which it is negligible in this experiment. However, constrained optimization still dominates unconstrained out-of-sample in our experiment for universe sizes up to 500 assets for both covariance treatments. Using the referee covariance in this experiment forces attenuating performance gains to be attributed to the basic failure of the law, rather than anything to do with covariance estimation, and the inversion of the performance relationship claimed by CST is maintained. These results should not be surprising; nor do they represent any serious contradiction to our basic thesis that adding securities adds little if any investment value, all other things the same.

## 4.4 Simulation Discussion

It should be stressed at this point that these experiments portray idealized versions of portfolio management, deliberately ignoring some important performance-impacting inevitabilities of practice. Uniformly high-quality information is free in this experiment, where in practice coverage of additional securities will always incur costs and is likely to degrade the overall quality (IC) of the system. Moreover, trading costs and other frictions associated with the practice of portfolio management are completely neglected in this analysis.

Our deliberate optimism in setting up the simulation has important implications. The unconstrained cases would likely exhibit poorer performance in practice. Since almost all of the assets in the simulation universe are likely to have some investment value, the investor is little harmed by putting portfolio weight on the "wrong" assets. In the real world, constraints will often limit the harm caused by misinformation. This effect was clearly demonstrated and measured in Jobson and Korkie (1981). In a truly chaotic world with a lot of estimation error and bias, the equal weighted portfolio, which uses no information to distinguish among assets, can be hard to beat.

[21] We note that the results reaffirm the conclusions in Frost and Savarino (1988).

Figure 1A: 0.10 IC, 50 Assets, Estimated Covariance

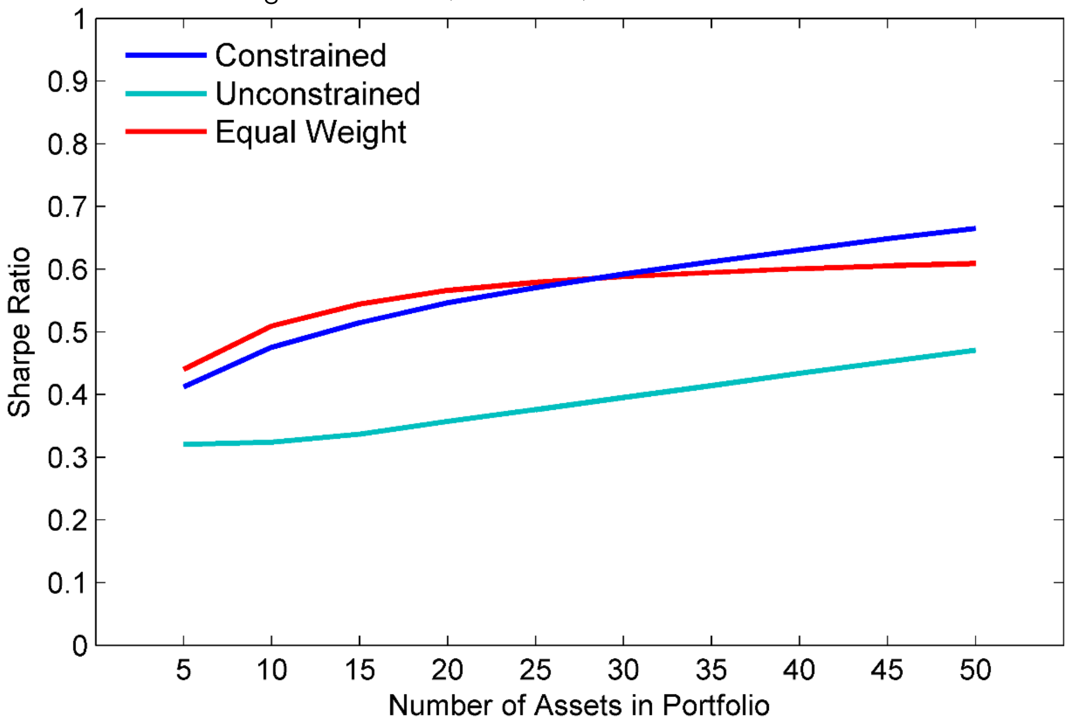


Figure 1B: 0.25 IC, 50 Assets, Estimated Covariance

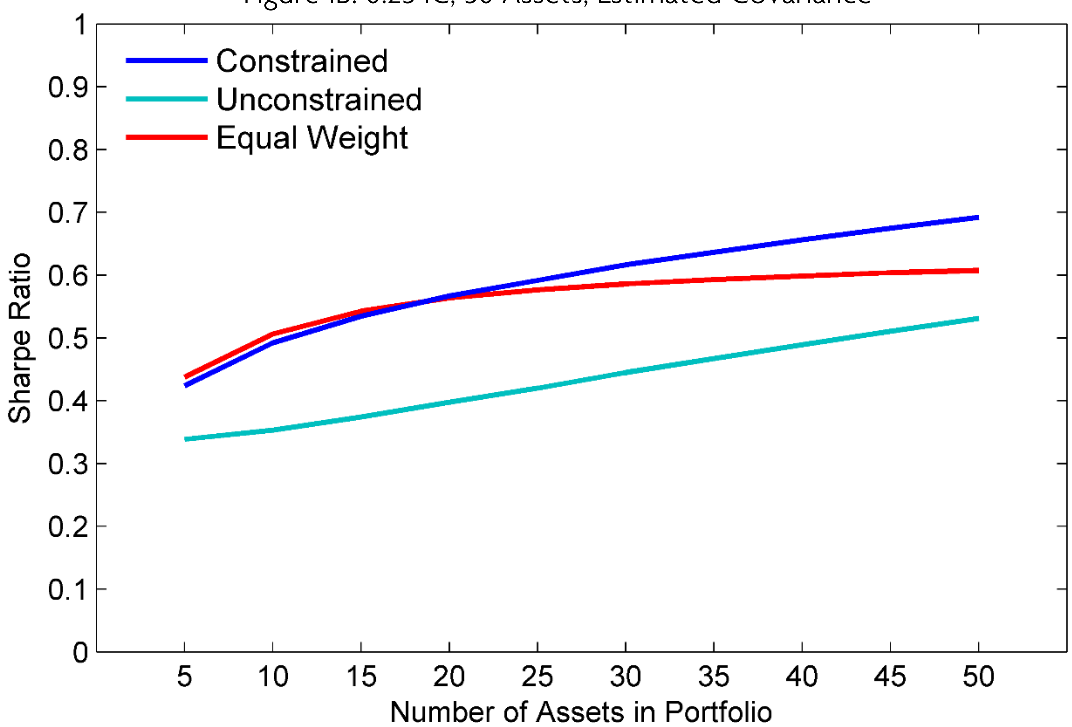


Figure 1: Average Sharpe Ratios for three different portfolio construction methods and two different information coefficients for the asset allocation case. This experiment was run on many simulations of up to fifty U. S. stocks which had at least 20 years of contiguous monthly price data ending in December 2013.

Note that our simulations maintain a constant level of IC independent of universe size, ignoring any realistic limitations on manager information. Larger universes, in our simulations, implicitly require larger levels of overall investment information from the manager, all other things the same. The slowly rising level of unconstrained average maximum Sharpe ratios as universe size increases is a necessary artifact of the simulation framework. In practice, adding assets is unlikely to add investment value beyond some optimal size universe consistent with the investor's level of information all other things the same. Indeed, beyond some optimal point, the unconstrained curve is likely to curve downward as the size of the optimization universe increases in applications. In practice, the accumulative costs associated with increasing the universe size will overcome the performance increases shown in our experiments, diminishing and eventually reversing performance gains. Increasing universe size with information of uncertain quality is never recommendable.

To summarize, our simulation experiment suggests several important conclusions. In real investment settings, an IC of 0.10 is often considered optimistic but nevertheless likely to be most relevant to practice of the cases within our study. We offer the following guidelines based on our experiments: 1) Equal weighting beats unconstrained optimization for realistically attainable information levels and is far less risky overall, for small optimization universes. The underperformance of unconstrained optimization relative to equal weighting is substantial enough to warrant general avoidance of unconstrained MV optimization in small universes. 2) Long-only constraints are likely to provide reliable performance gains over unconstrained optimization in many cases. 3) Universe size does not matter beyond a point, usually much smaller than the overall universe, and probably much smaller than much existing investment management practice. Increasing breadth past this saturation point, where the curves level off in the graphs, will likely provide no additional benefit, and may only incur unnecessary costs.

## 5.0 Some practical recommendations

This paper critiqued a number of ubiquitous and self-defeating strategies that have been proposed to add investment value to active asset management. In this section we provide some proposals and suggest some principals that have been useful in our own work in practice.

Merton (1987) counsels investing in what you know. But in practice, managers are often mandated to outperform an index while holding tracking error within a specified range. The Merton directive of investing in subsets of a large universe may often result in undesirably large tracking error. Michaud and Michaud (2005) provide a reconciliation of these competing objectives. They recommend including an index weighted "composite asset" representing the non-investment-significant alpha securities as part of the definition of the optimization universe. Adding the composite asset does not violate Merton's theoretical prescription while allowing for controlling tracking error risk in applications. Michaud and Michaud find that including a composite asset for non-investment significant assets often results in very desirable optimization and portfolio risk characteristics.

Figure 2A: 0.10 IC, 500 Assets, Estimated Covariance

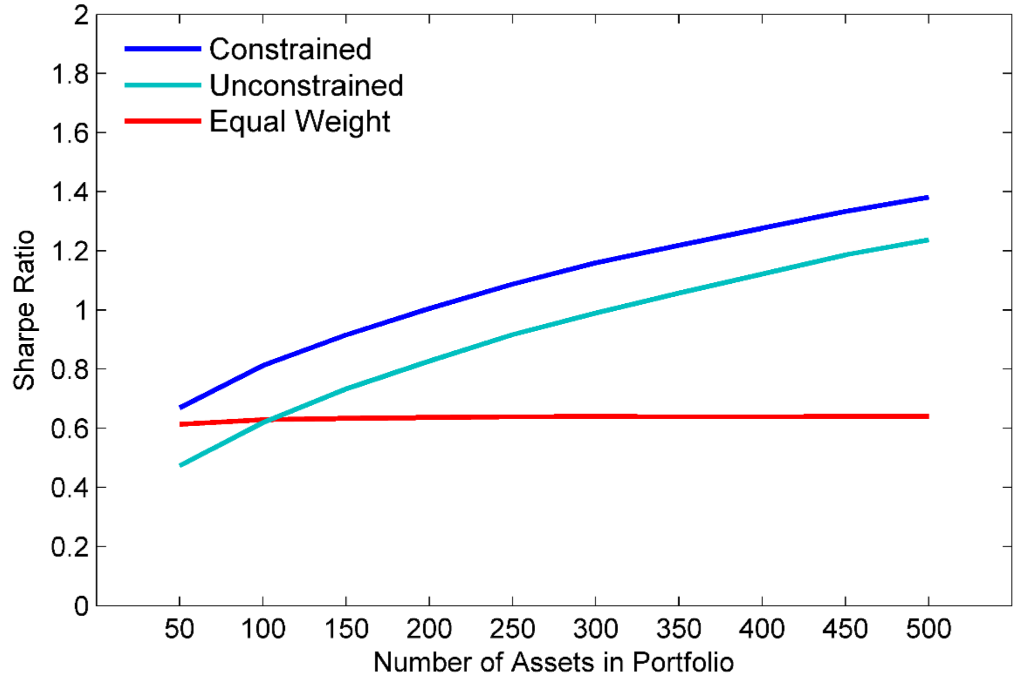


Figure 2B: 0.10 IC, 500 Assets, Referee Covariance (Error-Free)

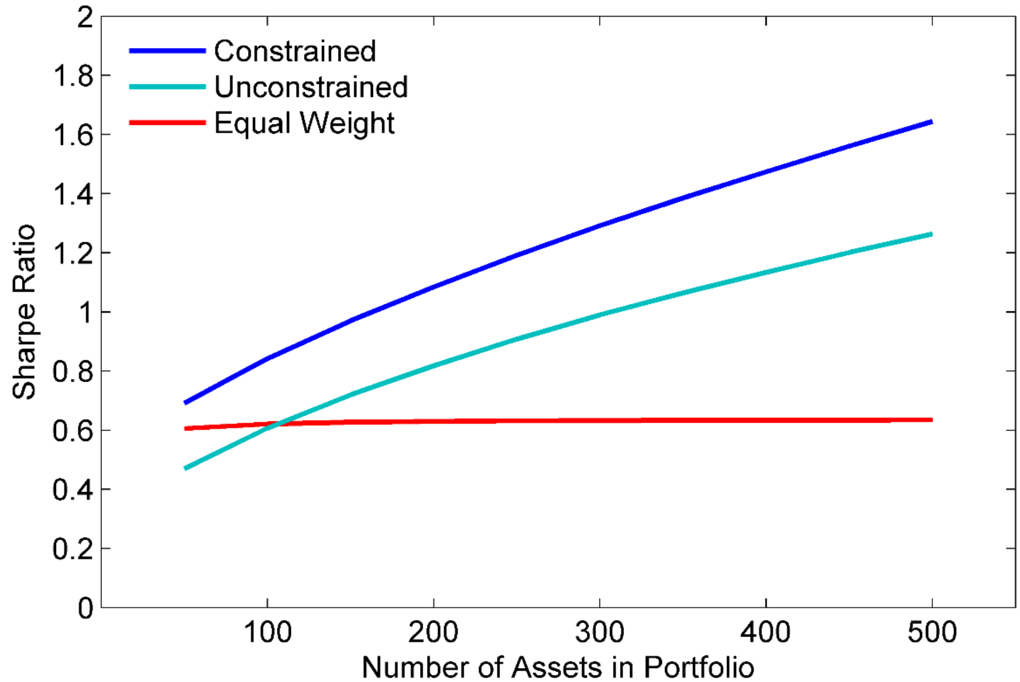


Figure 2: Average Sharpe Ratios for three different portfolio construction methods, using both estimated (2A) and referee (2B) covariance matrices in optimizations for the equity portfolio case, both with IC 0.10. This experiment was run on many simulated portfolios of up to five hundred U. S. stocks which had at least 20 years of contiguous monthly price data ending with December 2013. Figures 2A and 2B can be viewed as an extension of Figure 1A to larger optimization universes.

The appropriate number and character of factors that should be used for forecasting risk and return requires a number of considerations well beyond the scope of this report. However, some simple practical suggestions may be useful. Michaud (1990) shows that the presumed synergistic character of adding factors may have negative as well as positive consequences. Michaud (1998b) demonstrates that there are multiple kinds of "value" factors with different performance properties. Michaud (1999) provides a classification for understanding factor-return relationships in a "forward test" framework that notes variations relative to different global capital markets.

Trading in practice is typically calendar or range-rule based. Neither procedure is portfolio oriented or based on financial theory. Michaud el al (2012) notes that portfolio rebalancing is necessarily a statistical similarity test between the current and currently optimal portfolio. Resampling methods can be used to develop a statistically based rule for more efficient and effective trading.

Proper optimization constraints require many considerations. Regulatory and institutional constraints often dominate portfolio optimality structure in practice. Important economic considerations include the use of invested assets, competition, index structure and legal liability. What should not dominate are various marketing mandates that limit investment value.

Estimation error is a dominant factor in the investment value of optimized portfolios. Michaud optimization (Michaud, 1998; Michaud and Michaud 2008a, 2008b) is a generalization of the linear constrained Markowitz efficient frontier that includes estimation error in investment information in its portfolio construction methodology.[22] Monte Carlo sampling of risk-return estimates addresses uncertainty in investment information. An averaging process defines the new Michaud efficient frontier.[23,24] Simulation studies demonstrate that the resulting efficient frontier portfolios have superior investment value on average out-of-sample relative to Markowitz.[25]

## 6.0 Summary and conclusions

Our simulation studies confirm that the four investment strategy principles associated with the formula – frequent trading, large stock universes, adding forecast factors, and removing constraints – are invalid and generally self-defeating. The Grinold "law" is not a law of anything and does not help you invest better. Beyond some point, universe size and adding factors is irrelevant. Moreover, avoiding unconstrained MV optimization strategies is highly recommendable.[26]

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[22] Michaud resampled optimization was invented by Richard Michaud and Robert Michaud and is a U.S patented procedure, #6,003,018; worldwide patents pending. It was originally described in Michaud (1998, Ch. 6). New Frontier Advisors, LLC (NFA) is exclusive worldwide licensee.

[23] Uncertainty level can be defined by a "forecast confidence" scale based on estimation periods as described in Michaud and Michaud (2008a, 2008b) or by the investor's IC.

[24] We note that Michaud optimization is not the same as the Morningstar resampling optimizer which uses a different frontier averaging process with different out-of-sample investment properties. See Michaud and Esch (2010) for further information.

[25] Michaud (1998, Ch. 6), Michaud and Michaud 2008a,b. Markowitz and Usmen (2003) simulation studies indicate that Michaud optimization may be superior to Markowitz even with inferior risk-return estimates.

[26] Note that unconstrained utility function based optimization, ubiquitous in financial theory, is also subject to the same estimation error limitations documented in our simulations.

The necessary conditions for reliably winning the investment game include: 1) investment significant information for assets in an optimization universe; 2) economically meaningful constraints; and 3) properly implemented estimation error sensitive portfolio optimization technology.

The Grinold square root law is a theoretical construct that failed to consider the impact of estimation error on MV portfolio optimization. A nearly twenty-five year cohort of academic and practitioner research based on the formula and its extensions are not valid. Rationales for investing in many hedge fund, long-short, and unconstrained strategies are likely invalid. The implications of applications of the Grinold law may adversely impact a trillion dollars or more of AUM in current practice

Many formulas and laws in physics may fail to hold exactly but are supposed to hold somewhere. But the "Fundamental Law" fails not because it is an approximation but that it is based on a fundamentally flawed framework for practical asset management and should no longer be taught or used for practice.

The roots of the failure of the Grinold formula affect much of twentieth century finance, statistical science, and beyond. It is the result of the use of analytical methods and the assumption of known probability distributions similar to the certainty of a casino game. Grinold is simply one example in modern science of the fundamental and ubiquitous fallacy of regarding inference from in sample statistics and fixed probability models as the full measure of uncertainty.[27]

[27] Weisberg op. cit., p. xiii.

## References


Anson, M., D. Chambers, K. Black, H. Kazemi. 2009. *An Introduction to Core Topics in Alternative Investments* CAIA Association, 2nd ed. 2012. Wiley.

Buckle, D. 2004. "How to Calculate Breadth: An Evolution of the Fundamental Law of Active Portfolio Management." *Journal of Asset Management* 4(6):393–405.

Harvey, C., R. Liechty, R. Liechty 2008. "Bayes vs. Resampled efficiency: A Rematch." *Journal of Investment Management* 6(1):29-45.

Clarke, R, H. deSilva, S. Thorley. 2002. "Portfolio Constraints and the Fundamental Law of Active Management." *Financial Analysts Journal* 58(5): 48-66.

Clarke, R., H. deSilva, S. Thorley. 2006. "The Fundamental Law of Active Portfolio Management." *Journal of Investment Management*, 4(3): 54-72.

De Silva, H., Thorley, S., & Clarke, R. G. 2006. "The Fundamental Law of Active Portfolio Management." *Journal of Investment Management*, 4(3): 54-72.

Darnell, M. and K. Ferguson, 2014. "Thoughts On Grinold & Kahn's Fundamental Law Of Active Management" First Quadrant Perspective..

Diderich, C. 2009. *Positive Alpha Generation: Designing Sound Investment Processes*, Wiley.

Ding, Z. 2010. "The Fundamental Law of Active Management: Time Series Dynamics and Cross-Sectional Properties." Russell and Thaler Asset Management working paper.

Esch, D. 2015. "Some Notes on Information Correlation: Estimation and Simulation." Working paper, New Frontier Advisors.

Farrell, J. 1983. *Guide to Portfolio Management*. Wiley, NY.

Foquardi, S. and F. Fabozzi, 2004. The Mathematics for Financial Modeling and Asset Management, Wiley.

Frost, P. and J. Savarino. 1988. "For Better Performance: Constrain Portfolio Weights." *Journal of Portfolio Management* 15(1):29-34.

Gorman, L., S. Sapra,, R. Weigand. 2010. "The role of cross-sectional dispersion in active portfolio management." *Investment Management and Financial Innovations*, 7(3):54-64.

Grinold, R. 1989. "The Fundamental Law of Active Management." *Journal of Portfolio Management* 15(3): 30-37.

Grinold, R. and R. Kahn. 1995. *Active Portfolio Management: Quantitative Theory and Applications*. Chicago: Irwin.

Grinold, R. and R. Kahn, 1999. *Active Portfolio Management: A Quantitative Approach for Providing Superior Returns and Controlling Risk*. 2nd ed. New York: McGraw-Hill.

Huij, J. and J. Derwall. 2011. "Global Equity Fund Performance, Portfolio Concentration, and the Fundamental Law of Active Management." *Journal of Banking & Finance*, 35(1):155-165.

Jacobs, B. and K. Levy, 2008. *Investment Management: An Architecture for the Equity Market*. Wiley

Jobson, D. and B. Korkie, 1981. "Putting Markowitz Theory to Work." *Journal of Portfolio Management* 7(4): 70-74.

Kahn, R. 1997. "The Fundamental Law of Active Management: Seven Quantitative Insights into Active Management—Part 3," BARRA Newsletter Winter.

Kroll, B. D. Trichilo, and J. Braun, 2005. "Extending the Fundamental Law of Investment Management (or How I Learned to Stop Worrying and Love Shorting Stocks)." JP Morgan Asset Management white paper.

Ledoit, O., 1997 "Improved Estimation of the Covariance Matrix of Stock Returns with an Application to Portfolio Selection." Working paper. Anderson Graduate School of Management, University of California at Los Angeles.

Ledoit, O., and Wolf, M. 2004a. "A Well-Conditioned Estimator for Large-Dimensional Covariance Matrices". *Journal of Multivariate Analysis*, 88(2): 365-411.

Ledoit, O., and Wolf, M. 2004b. "Honey, I Shrunk the Sample Covariance Matrix". *Journal of Portfolio Management*, 30(1):110-119.

Markowitz, H. 1952. "Portfolio Selection." *Journal of Finance* 7(1): 77-91.

Markowitz, H., 1959. *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley and Sons. 1991. 2nd ed. Cambridge, MA: Basil Blackwell.

Markowitz, H. 2005. "Market Efficiency: A Theoretical Distinction and So What?" *Financial Analysts Journal* 61(5): 17-30.

Markowitz, H. and N. Usmen, 2003. "Diffuse Priors vs. Resampled Frontiers: An Experiment." *Journal of Investment Management* 1(4): 9-25.

Merton, R. 1987. "Presidential Address: A Simple Model of Capital Market Equilibrium with Incomplete Information." *Journal of Finance* 42(3): 483-510.

Michaud, R. 1989. "The Markowitz Optimization Enigma: Is Optimized Optimal?" *Financial Analyst Journal* 45(1):31-42.

Michaud. R. 1990. "Demystifying Multiple Valuation Models." *Financial Analysts Journal* 46(1):6-8.

Michaud, R. 1993. "Are Long-Short Equity Strategies Superior?" *Financial Analyst Journal* 49(6):44-49.

Michaud, R., 1998. *Efficient Asset Management*. New York: Oxford University Press, 2001. First published by Harvard Business School Press.

Michaud, R., 1998b. "Is Value Multidimensional? Implications for Style Management and Global Stock Selection." *Journal of Investing*, 7(1):61-65.

Michaud, R. 1999. *Investment Styles, Market Anomalies, and Global Stock Selection*. Research Foundation of the Chartered Financial Institute, Charlottesville.

Michaud, R. and R. Michaud. 2005. "Equity Optimization Issues – III: Insignificant Alphas and Heterogeneous Errors." Newsletter: New Frontier Advisors, Boston, March.

Michaud, R. and R. Michaud. 2008a. *Efficient Asset Management*. 2nd ed. Oxford University Press: New York.

Michaud, R. and R. Michaud 2008b. "Estimation Error and Portfolio Optimization: A Resampling Solution." *Journal of Investment Management* 6(1):8-28.

Michaud, R. and R. Michaud 2008c. "Discussion on article by Campbell R. Harvey, John C. Liechty and Merill W. Liechty, Bayes vs. Resampling: A Rematch (2008, Volume 6, Number 1), *Journal of Investment Management* 6(3):1-2.

Michaud, R. and D. Esch, 2010 "Morningstar vs. Michaud Efficient Frontiers." New Frontier Advisors Research Paper.

Qian, E and R. Hua., 2004. "Active Risk and Information Ratio." *Journal of Investment Management*, 2(3):1-15.

Roll, R. 1992. "A Mean/Variance Analysis of Tracking Error." *The Journal of Portfolio Management*, 18(4):13-22.

Schulmerich, M., Y. Leporcher, and C. Eu. 2015. *Applied Asset and Risk Management,* Springer: Heidelberg

Utermann, A. 2013. "The Changing Nature of Equity Markets and the Need for More Active Management" Allianz GI Global Capital Markets & Thematic Research.

Weisberg, H. 2014. *Willful Ignorance: The Mismeasure of Uncertainty*. Wiley: New York.

Zhou, G. 2008. "On the Fundamental Law of Active Portfolio Management: What Happens If Our Estimates Are Wrong?" *Journal of Portfolio Management,* 34(4):26-3