

WORLD GOLD COUNCIL

Gold as a Strategic Asset

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September 2006

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September 2006

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NFA Company profile

New Frontier Advisors, LLC (NFA) is an institutional research and investment advisory firm specialising in the development and application of state-of-the-art investment technology. Based in Boston, NFA provides consulting and investment advisory services as well as licensing of patented and proprietary software. NFA principals invented the world's first broad spectrum, provably effective, portfolio Optimisation, rebalancing, and monitoring process. The Resampled Efficient Frontier™ optimiser is globally recognised as a landmark development for asset allocation and equity portfolio management.

Through monographs, refereed academic and professional papers, patents, white papers, seminars, and invited presentations, NFA continues to pioneer new developments in portfolio management, investment strategy, and financial planning tools. The firm specialises in state-of-the-art applications of contemporary financial theory, mathematical statistics, and computer science. NFA principals have over 80 years of institutional experience in consulting, asset management, financial planning, financial research, and investment technology development. NFA combines practical investment experience, patented techniques, and world class research and management skills to offer uniquely effective institutional quality investment services.

World Gold Council – Corporate Profile

Founded in 1987, the World Gold Council, the marketing organisation formed and funded by the world's leading gold mining companies, represents 24 companies and around 40% of total gold production. The World Gold Council is an international, not for profit organisation, with offices in India, China, Japan, the Middle East, Turkey, Western Europe and North America. The World Gold Council is the gold industry's key marketing body. We work closely with jewellery retailers, manufacturers, wholesalers, banks, investment companies and distribution specialists to promote the use of gold in all its forms, be it jewellery, investment or industrial applications.

Our work in the investment sector focuses on three core areas: research, communication and facilitating gold investment by improving the ease of access. We believe that gold as an investment is as relevant in the 21st century, as it has been for hundreds and even thousands of years. The World Gold Council does extensive work on communicating the investment case for gold to investors. We have built up a body of research that is highly regarded and used, by pension fund advisors, fund managers, precious metals analysts, private client advisors and central banks. In addition, we have developed good working relationships with traditional gold market participants, as well as strategists and investment consultants seeking robust, independent information about gold investment to help form decisions about making long-term allocations to gold in client portfolios.

The World Gold Council is independent from promoting any specific form of gold investment, but is informed on all methods. There are a number of alternative ways to invest in gold including bullion coins and bars, exchange-traded products or special funds. The attractiveness of each of these depends on a number of factors. Does the investor want to own gold or does he simply want exposure to gold price fluctuations? Is the investor comfortable with the idea of leverage and margin calls or not? Does the investor understand the fee structures attendant upon each type of product? Regulatory constraints may also restrict access to certain types of investment and this is something else to take into consideration. These apply regardless of the particular set of reasons driving an investment strategy.

The World Gold Council provides extensive information for investors – our website, www.gold.org, is the prime medium for publishing this research and statistics, as well as providing fairly exhaustive information about gold.

As part of our investment research and marketing programme, we conduct regular briefing sessions on the gold market for investment professionals in Switzerland, United Kingdom and the United States.

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In addition to analysis of gold market fundamentals with respect to their impact for investors, Katharine's core research area is the strategic role of gold in investment portfolios.

Before joining the World Gold Council, Katharine was working on a PhD in Economics at the School of Oriental and African Studies, University of London, whilst teaching Econometrics and Finance. She obtained her MSc in Economics (with distinction) from the same college and also holds a degree in Psychology and Sociology from the University of Cape Town.

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Executive Summary

We examine the case for gold as a long-term or strategic investment for U.S. institutional investors. The role of gold in asset management is currently very topical. Much of the interest, however, is related to short-term issues such as hedging the value of the dollar. From a longer term perspective a fairly wide consensus exists that gold retains inflation-hedging properties despite considerable fluctuations in the shorter term. Prior to the early 1970s, gold's price was fixed to the dollar and its investment value was necessarily limited. There now exists more than thirty years of return history to evaluate gold as a strategic asset. Earlier studies that reported favourable evidence for the investment value of gold were generally limited by shorter term return history, not always relevant index comparisons, unsophisticated statistical estimation techniques and unstable and ineffective Optimisation frameworks. We use state-of-the-art statistical estimation technology based on current empirical data and conservative estimates and Resampled Efficiency™ (RE) Optimisation to avoid the limitations of conventional asset allocation technology and for statistically analysing the significance of gold. Our results show that gold may have a comparable portfolio weight to asset classes such as small cap and emerging markets due to its value as a diversifying asset class. A strategic allocation to gold is dependent on portfolio risk level. We find a small though significant allocation of 1 to 2% at low risk and 2 to 4% in a balanced portfolio. While not statistically significant at high risk levels, gold may provide stability in poor markets and economic climates to long-term institutional strategic investors.

Introduction

Despite centuries' worth of fascination with gold, and rising interest of late, current investment interest in gold depends on investor habitat. Short-term tactical investors have an intense interest as geopolitical events and unstable global currencies may drive gold price volatility. Intermediate-term investors cite higher demand for gold for jewellery and industrial applications combined with an inelastic supply that may take several years to catch up as their impetus for investment, resulting in a moderate interest. However, long-term strategic investors, do not, as a rule, invest in gold. This paper examines the case for gold as a long-term, strategic investment for U.S. institutional investors. We use simulation studies, empirical analysis, and economic theory to determine the appropriate allocation of gold in a strategic portfolio.

Prior to the early 1970s, gold's price was fixed to the dollar and its role in asset management was necessarily limited. Between 1933 and 1974 private sector investors in the United States were prohibited from owning gold. There now exists more than thirty years of return history for evaluating gold's role in an institutional strategic asset allocation. Improved statistical techniques such as resampling for capturing more information in data and estimating risk-return more reliably are now available.¹ We use Resampled Efficiency™ (RE) Optimisation to avoid the limitations of conventional asset allocation technology and for statistically analysing the significance of gold.² Our results show that gold can be a significant component of a strategic asset allocation for long-term institutional investors.

A Brief History of Gold

Gold has been used as money to a greater or lesser extent for much of the history of civilisation. Under the international gold standard that existed for much of the 19th century until World War I, currency was backed by gold, which provided a “good housekeeping seal of approval.”³ After World War II, the convertibility of the dollar into gold at a fixed rate of \$35 per troy ounce underpinned the stability of the new financial order set up at Bretton Woods in 1945. By the late 1960s, inflationary pressures were growing untenable and it was becoming clear that the dollar would need to devalue relative to gold and, by implication, other currencies. In 1968 a two-tier system was set up, with a free private market in gold, while central banks continued to transact among themselves at the official rate. By 1971 the pressure on the dollar which was still convertible into gold could no longer be sustained and the “dollar-gold” window was closed at \$42.22 per troy ounce. The chapter of fixed exchange rate regimes, including a fixed gold price, had come to an end. As Figure 1 shows, the new world of floating exchange rates, oil supply shocks, the effect of the silver corner during the late 70s and early 80s, and other issues resulted in a substantial increase in price soon after the gold standard was lifted and, until recently, this was followed by a relatively narrow trading range.

¹ See Efron and Tibshirani (1993) for more detailed information. See also Efron (2005) for an up-to-date discussion of applications in a wide variety of areas by the inventor of the idea.

² Invented by Richard Michaud and Robert Michaud. U.S. patents 6,003,018, 6,928,418, Israel 138018. Worldwide patents pending. New Frontier Advisors LLC is worldwide licensee.

³ Bordo and Rockoff (1996).

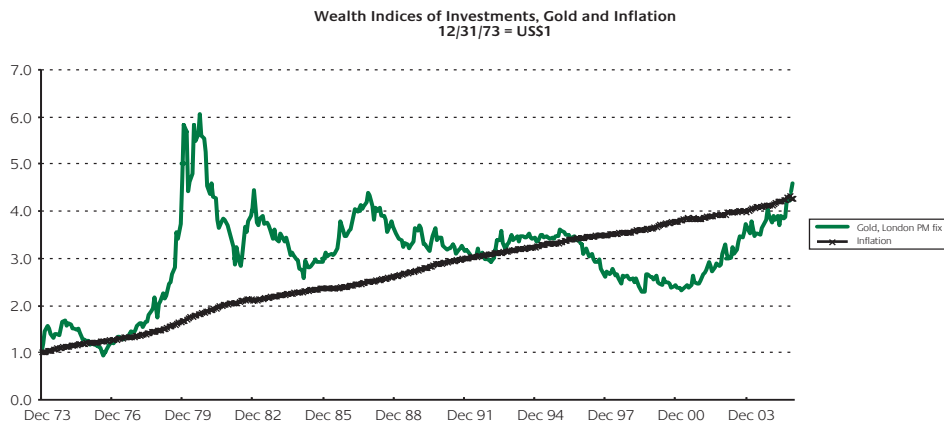
Figure 1: Gold Price 1968 - 2005



The Investment Value of Gold

Gold is often thought to be an inflation hedge for U.S. investors. Figure 2 displays the relationship between gold and the Consumer Price Index (CPI) from 1974 to 2005. While gold and the CPI performed similarly over the entire period there were many subperiods where performance was unrelated. Very long term evidence, from 1802 to 2001 (Siegel 2002), shows a similar pattern. Levin and Wright (2006) provide a model relating short run deviations to long term real value.⁴ The long-term relationship between gold and inflation provides a basis for forming the conservative return expectation that the real return on gold should be equal to zero which is equivalent to stating that gold price is roughly constant in real terms over the very long run.

Figure 2: Gold vs Inflation 1974 - 2005



The dynamics and structure of gold supply and demand may underlie the lack of correlation between returns on gold and returns on other assets. At a minimum, gold

⁴ See also Burton et al (2006) and Harmston (1998).

continues to play a dual role as a monetary asset or investment and as a commodity. Monetary or investment demand for gold has tended to dominate during periods of global economic and geopolitical turmoil but is also a significant perennial factor in certain countries with less developed banking systems and historically high inflation.

The purest commodity demand for gold derives from its medical and industrial uses. This demand is driven by similar economic factors to those that determine the use of other commodities as inputs, including the availability or desirability of substitute materials.

Since the end of the gold standard era over thirty years ago, some 75% of gold purchased has been in the form of jewellery. The drivers of jewellery demand are complex and vary widely from country to country. For example, market research carried out in 2005 revealed 69% of respondents in the key gold consuming countries of China, India, Italy, Saudi Arabia, Turkey and the USA purchased gold jewellery as much as an investment as a fashion item.⁵ As a luxury good, the demand for gold jewellery appears more responsive to fluctuations in income than to price levels, although in certain countries demand often displays a strong sensitivity to the rate of change in the gold price. The geographical diversity of gold demand is another aspect underpinning price movements that tend to be independent of those of the main capital markets.

The long lead times, typically in excess of five years, that constrain the responsiveness of gold supplied through mine production (around three fifths of annual supply), coupled with international agreements regulating sales by central banks, mean that the main source of supply elasticity is recycled gold. Surges in supply from this source have historically been related to specific instances of distress selling in response to localised economic crisis.

This brief overview sheds some light on the economic drivers that may underpin gold's ability to provide risk-reducing diversification with respect to other assets.⁶

Strategic Asset Allocation

The importance of strategic asset allocation is primarily based on the pioneering Brinson et al (1986, 1991) studies. Using a historical return database of large pension plans they found that the main explanatory performance variable is average risk policy or the stock/bond mix; this one factor explained nearly 94% of the variance in performance. The two other explanatory components of performance – tactical asset allocation or market timing and security selection – were of marginal importance and negatively related to return. Their results are the basis for much of the consensus in the investment community that the average long-term risk policy is the single most important investment decision.⁷

⁵ A Passion for Gold: Realising Potential in the Gold Market. Consumer Research Summary, World Gold Council, 2005. http://www.gold.org/pub_archive/pdf/a_passion_for_gold.pdf

⁶ For further discussion of the structure and dynamics of the gold market, see www.gold.org/value/markets/supply_demand/index.html

⁷ An important alternative study that also used an empirical database of the performance of large pension plans was Hensel et al (1991). They found that risk policy was roughly equally important with tactical and security selection.

The strategic importance of an asset relative to a given set of asset classes and risk policy is traditionally evaluated in the context of asset allocations for optimised portfolios on the Markowitz (1959) mean-variance (MV) efficient frontier. The optimised allocations are presumed to be a measure of the asset's importance and role in improving portfolio reward-to-risk.

Strategic Allocation Empirical Studies

There have been some studies that have found an important role for gold or its surrogates in a strategic asset allocation. Such studies are almost always based on a relatively large return premium for gold. For example Jaffe (1989) finds a significant role for gold in a MV optimised portfolio. However, his historical data estimates a real return for gold of more than 12%.⁸ A more recent study by Idzorek (2005) has a similar empirical character. He finds a return premium of 6.3% relative to inflation for an index equally-weighted in gold, platinum, and silver and time period 1972 to 2004. In contrast the real return premium for gold for the period of our study, from 1974 through 2005, is 2.1%, a far more believable number. It is not a difficult matter to make a case for the strategic benefit of gold if we assume a 6% or more inflation adjusted return premium. But such assumptions are unlikely to be reliable or useful for long-term strategic asset management.

Commodity Indices

Strategic asset allocation typically focuses on asset classes rather than individual assets. For this reason institutional investors may consider that investment in gold should properly be considered in the context of a basket of well diversified investable commodities. While investing directly in a basket of underlying commodities may seem conceptually attractive, it is a route that in practice exists only for the precious metals gold, silver and platinum. Given size of markets, liquidity issues, and relative homogeneity, a basket of precious metals may have little additional institutional attraction relative to gold. The question of interest is what is to be gained from investing directly in gold over a well diversified basket of commodities.

For reasons of commodity heterogeneity, including delivery, storage, and durability or its lack, interest in diversified baskets of commodities is generally associated with commodity index futures. The three most popular indices for U.S. institutional investors are the Goldman Sachs Commodity Index (GSCI), the Dow Jones-AIG Commodity Index (DJ AIG) and the Reuters-CRB (Commodity Research Bureau) Index (CRB).⁹ Each index is weighted differently. For example the GSCI is heavily weighted towards energy while the CRB is more equally weighted. The GSCI has the most open interest followed by the DJ AIG and CRB. Each represents a very broad spectrum of commodity investment opportunities. For example, the GSCI index invests in 24 futures contracts; the DJ AIG invests in 20 and the CRB in 17.

⁸ Jaffe's data runs from September 1971 to June 1987. Over this time period gold was very nearly the best performing asset.

⁹ Note that the Reuters-CRB Index was renamed the Reuters/Jefferies CRB Index in 2005.

Gorton and Rowenhorst (2006) report very attractive commodity index investment performance: low correlations with standard asset classes and risk-returns comparable to equity indices. Weighting schemes as well as composition can be responsible for very different index performance in a given period. Responding to critiques of some commercial index weighting schemes, Gorton and Rouwenhorst (2006) construct an equal-weighted index back to 1959 covering 34 commodities. They found that commodity futures perform similarly to the S&P 500 index and are significantly superior to bonds. In our study we use the CRB index because it is the most closely equal weighted of the commercial indices and has the fewest commodity components.

Commodity Futures Indices and Gold

What is the relevance of commodity future index returns for understanding gold? Foresti and Toth (2005) review the five components of commodity futures return. These are: 1) insurance or risk premium; 2) collateral yield; 3) rebalancing yield; 4) roll or convenience yield; 5) expectational variance.

Risk premiums are associated with producers separating business risk from commodity price risk. Given the wide variety of business risks and properties of commodities in the commodity futures indices, the risk premium is unlikely to greatly resemble the risk premium for gold. Collateral yield is associated with the mechanics of a futures trade; no transfer of cash is required to initiate the investment. Investors retain the use of capital and the return is earned in addition to the return on investing in the commodity futures. Rebalancing yield and weighting schemes may result in a commodity index growing in value even if on average the components do not. Erb and Harvey (2006) describe the process as “converting water into wine.” Roll or convenience yield has to do with re-investment in futures contracts that subsequently come due.¹⁰ Expectational variance is the difference between the expected and actual spot price. Expectational variance includes changes in unexpected inflation as well as changes in the economic value of the commodities.

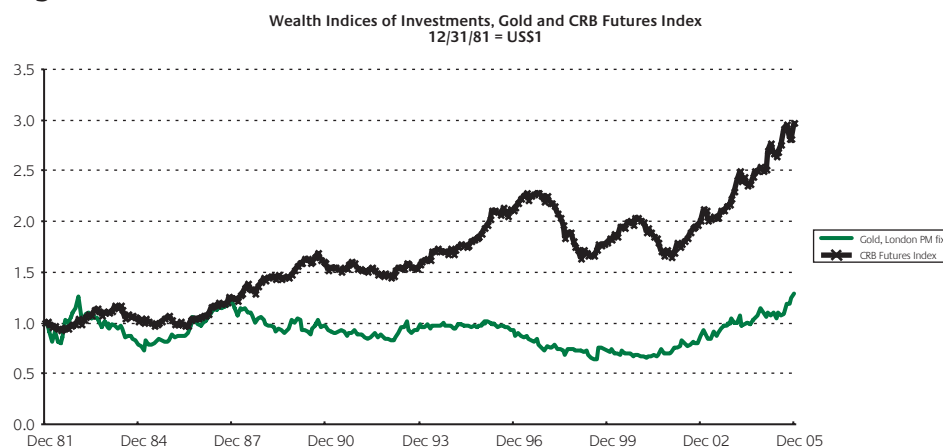
Nearly all the components of commodity future index returns are irrelevant for understanding gold price return. Gold is a minor factor in the performance of the commodity futures indices. Commodity futures indices are often not very transparent while a gold price return index is a transparent and direct measure of the object of interest.

Figure 3 displays the performance history of gold relative to the CRB index from 1982 to 2005. Empirical data strongly confirms the limitations of studying gold in the context of commodity index futures. While we shall include the CRB commodity index for comparison purposes, our focus is on the risk-return characteristics of the gold price return index relative to assets of interest for institutional strategic allocation. Whatever

¹⁰ Gorton and Rouwenhorst report that between 1959 and 2004, the historical risk premium (excess return) of their equally weighted, fully collateralised basket of commodity futures averaged 5.3% per year. However, given that the average yield on three month Treasury bills from 1954 to 2004 was 5.2%, this implies that the net average contribution of roll yield was 10 basis points.

the investment merits of commodity indices, they are not a useful surrogate for understanding the importance of gold in a strategic asset allocation. Although investor interest in commodities is growing and evolving rapidly, relatively few institutional investors have made strategic allocations to date.

Figure 3: Gold vs CRB Futures Index 1982 - 2005



Study Framework

Our study of gold as a strategic asset is based on five sets of asset allocation studies. The cases span a spectrum from basic to expanded asset classes, from long-term risk-return estimates to first principle conservative estimates, and some additional recent time period studies.¹¹

Since gold and commodities are generally assumed to have inflation hedging character, all historical risk-return estimates are inflation adjusted. Gold was pegged to the U.S. dollar for many years in the twentieth century. Pegged dollar returns are unlikely to be useful in a contemporary asset allocation study. Our monthly total return data begins safely after the dollar peg, from January 1974 to December 2005.

Some indices were not available throughout the entire term of the study period. For example the Russell equity indices were available from 1979 and the CRB from 1982. The historical risk-returns in the study use the EM algorithm to consistently and rigorously fill in the effect of missing data over the period and avoid the many ad hoc practices associated with incomplete return series assets in a portfolio Optimisation.¹² Our tests have indicated that the EM algorithm works well for estimating risk-return estimates with missing data for many asset allocation applications. The EM algorithm is used throughout the study.

¹¹ Descriptions of the monthly return series for the Consumer Price Index (CPI), T-bills, intermediate-term and long-term government bonds are given in Ibbotson (2005, Ch. 3). Large and small capitalisation equity indices are from the Russell 1000 and 2000 equity indices; international developed equities from Morgan Stanley Capital Perspective Europe Asia and Far East (EAFE) index.

¹² See Carlin and Louis (1996) for further details of the EM algorithm.

The asset allocation studies use RE Optimisation, a provably effective enhancement of MV Optimisation.¹³ Three different sets of optimised asset allocations are performed: with or without gold and with gold and the CRB. The Ledoit (1997) procedure for improved covariance estimation is used in all studies except the base case.¹⁴ Non-negativity and budget constraints are imposed on the Optimisation. RE Optimisation is described in more detail below.

Case 1: The Base Case

A minimal set of asset classes consistent with contemporary institutional strategic asset allocation practice includes: T-bills, intermediate and long-term fixed income, large and small capitalisation domestic equities and developed market international equities. The inflation-adjusted average returns, standard deviations, and correlations are given in Table 1. The EM algorithm was used for the Russell and CRB indices.

Case 2: Strategic Premiums

Historical risk-return estimates generally reflect some ephemeral effects. Period dependency is inconsistent with the purpose of a strategic asset allocation study. Our objective is not to measure what happened over some particular time period but to estimate what is likely to happen going forward over the long term.

One simple solution to minimise period dependency is to use very long-term historical return data. But even long-term data is likely to have period dependent effects. For example, U.S. long-term historical risk-return estimates reflect the performance of the most successful world capital market, a condition not guaranteed to persist. An alternative approach, exemplified in Case 2 and 3, is to use conservative return premium assumptions for gold, commodities, and other assets. The more conservative the assumptions the more likely the significant results are reliable.

Table 2 summarises our strategic return premium assumptions for the assets in the base case studies. T-bill inflation adjusted return for much of the twentieth century is less than 1%. There were many changes in monetary policy relative to interest rates during this period. As a conservative estimate, a zero real return for T-bills seems reasonable; if the assumption is in error it is unlikely to affect our results in any significant way. Following Siegel (2002) we assign a zero real return to gold. Since gold is the primary focus of our study, assigning a zero real return will free our results from any special pleading bias

¹³ RE Optimisation is not to be confused with the Feldman (2003) procedure used in Idzorek (2005). RE Optimisation is patented with very important fundamental differences in investment properties as well as proofs of superior performance. See Michaud and Michaud (2006) for more information.

¹⁴ The sample covariance matrix estimate is known to be error prone. Therefore, we use Ledoit's empirical Bayes estimator of the covariance matrix. Ledoit uses a shrinkage parameter to pull the extreme high and low covariance elements toward a more sensible central belief. We use the concept of market equilibrium to determine the values of this central belief. The result is a more robust covariance matrix less sensitive to extreme observations and spurious correlations. See also Michaud (1998, Ch. 8) for additional discussion.

Table 1 : Basic Assets: Inflation-Adjusted Risk-Return Estimates January 1974 - December 2005

Asset Names	Return	Std dev	Correlations										
			US Tbilis	Intrmd US Gov Bonds	Long Term US Govt Bonds	Russell 1000 Large Cap US Equity	Russell 2000 Small Cap US Equity	MSC EAFE International Equity	CRB Futures Index	Gold, London PM Fix			
US Tbilis	1.4%	1.1%	1.00										
Intermediate US Govt Bonds	3.6%	6.1%	0.35	1.00									
Long Term US Govt Bonds	4.9%	10.8%	0.29	0.86	1.00								
Russell 1000 Large Cap US Equity	9.3%	15.5%	0.16	0.20	0.26	1.00							
Russell 2000 Small Cap US Equity	9.8%	19.8%	0.12	0.11	0.15	0.84	1.00						
MSCI EAFE International Equity	7.2%	17.0%	0.17	0.16	0.18	0.61	0.56	1.00					
CRB Futures Index	2.5%	9.7%	-0.09	-0.13	-0.10	0.14	0.18	0.26	1.00				
Gold, London PM Fix	2.1%	19.7%	-0.15	0.02	0.00	0.07	0.14	0.20	0.59	1.00			

*Russell from January 1979, CRB from January 1982.
EM and Ledoit estimated

critique. Note that the standard error of the mean for gold of 3.5% implies that zero real return is within the standard error confidence interval for Table 1 data.¹⁵ Finally, a zero real return for an equal-weighted basket of commodities seems reasonable from a strategic perspective and consistent with our focus on gold in the context of more traditional strategic asset classes.

Table 2: Basic Assets: Inflation-Adjusted Risk-Return Estimates Strategic Return Premiums January 1974 – December 2005

Asset Names	Return	Std dev
US TBills	0.0%	1.1%
Intermediate US Govt Bonds	3.6%	6.1%
Long Term US Govt Bonds	4.9%	10.9%
Russell 1000 Large Cap US Equity	9.3%	15.4%
Russell 2000 Small Cap US Equity	9.3%	19.6%
MSCI EAFE International Equity	9.3%	17.0%
CRB Futures Index	0.0%	9.5%
Gold, London PM Fix	0.0%	19.7%

*Russell from January 1979, CRB from January 1982
EM and Ledoit estimated

The return premiums for the remainder of the assets are more or less based on our historical risk-return estimates for the period. More specifically, we use the historical average inflation adjusted returns of 3.6%, 4.9%, and 9.3% from Table 1 for intermediate, long-term government bonds and large cap equities strategic return premiums.¹⁶ The large cap equities return assumption is statistically consistent with Siegel's (2002) 8.9% for the 1926 to 2001 period. Following Michaud et al (1996) we assign the same return premium of 9.3% to international as large cap U.S. equities. We also assign the same 9.3% return premium to small cap domestic equities. As in Table 1, empirically small cap stocks have exhibited a positive return premium relative to large cap. But the actual observed premium can be very dependent on the small cap index used as well as time period studied.¹⁷ Given our bias for conservatism and the fact that gold is the major focus of our study, such an assumption should have little impact on our results.

Estimation error in correlations is far more serious than commonly assumed. The Ledoit (1997) estimator is a simple reliable procedure for improving the forecast investment value of correlations in our optimisations.

¹⁵ Gold is the only asset in Table 1 such that zero is in the standard error confidence interval.

¹⁶ The return data for U.S. T-Bills, intermediate government bonds, and long-term government bonds from Ibbotson Yearbook (2006); MSCI EAFE (Europe Australia Far East) index from Morgan Stanley (2006); Russell 1000 and Russell 2000 Total Return indices from Morningstar (2006); CRB index from Commodities Research Bureau (2006); London Gold Spot data from World Gold Council.

¹⁷ For example the Ibbotson Associates small cap index over the 1974 to 2005 period has roughly a 5% premium over large cap, whereas the Russell index premiums are closer to 2% or less. Also the results are often very time period dependent.

Case 3: Expanded Asset Classes with Strategic Premiums

Institutional strategic investors often include a number of additional asset classes beyond those in our base case studies. We include long-term corporate bonds, high-yield bonds, non-U.S. bonds, emerging markets, and REITs in the expanded asset class studies. This more comprehensive set of assets is a fairly close approximation to contemporary institutional strategic asset allocation practice.¹⁸

Table 3 provides a summary of the strategic return premiums associated with the additional asset classes in the expanded asset studies. Adding more assets in a strategic asset allocation implies more estimation error in the Optimisation process and the possibility of more period dependent results. In order to minimise the effect of estimation error and period dependency on our results we assign conservative strategic premiums to the additional five assets. All three new bond assets, long-term corporate, high-yield, and non-U.S. bonds are assigned the same premium as long-term government bonds. Emerging markets are assigned the same premium as large cap domestic and international equities. REITS are a hybrid security having characteristics of fixed income instruments and equities. For simplicity we assume a strategic return halfway between large cap and long-term government bonds.

Table 3: Expanded Assets: Inflation-Adjusted Risk-Return Estimates Strategic Return Premiums January 1974 – December 2005

Asset Names	Return	Std dev
US TBills	0.0%	1.1%
Intermediate US Govt Bonds	3.6%	6.1%
Long Term US Govt Bonds	4.9%	10.8%
Long Term US Corporate Bonds	4.9%	9.7%
High Yield US Corporate Bonds	4.9%	8.4%
World ex US Bonds	4.9%	9.5%
Russell 1000 Large Cap US Equity	9.3%	15.5%
Russell 2000 Small Cap US Equity	9.3%	19.8%
MSCI EAFE International Equity	9.3%	17.0%
MSCI Emerging Markets Equity	9.3%	24.0%
DJ Wilshire REIT Real Estate	7.1%	15.0%
CRB Futures Index	0.0%	9.7%
Gold, London PM Fix	0.0%	19.7%

*Russell from January 1979, CRB from January 1982, High Yield from July 1983, ex-US Bonds from January 1978, Emerging Equity from January 1988, REITS from January 1978. EM and Ledoit estimated

¹⁸ Additional asset indices: long term corporate bond data from Ibbotson Yearbook (2006); American corporate high yield bond data from Lehman Brothers (2006); Global ex-U.S. bond data from Citigroup World GVT Bond Index unhedged total returns from 1985 and supplemented with Lehman Eurobond Global Index available from 1978 in Michaud (1998); MSCI Emerging from Morgan Stanley (2006); Dow Jones Wilshire REIT from Wilshire Associates (2006).

Case 4: Returns from 1986

Table 4 provides a summary of the historical risk-return estimates for the last twenty years for the expanded set of assets. Among other reasons for interest, this time period avoids the late 70s and early 80s when the silver corner influenced gold prices.

Table 4: Expanded Assets: Inflation-Adjusted Risk-Return Estimates January 1986 – December 2005

Asset Names	Return	Std dev
US TBills	1.5%	1.0%
Intermediate US Govt Bonds	4.3%	4.8%
Long Term US Govt Bonds	6.8%	9.7%
Long Term US Corporate Bonds	6.3%	7.6%
High Yield US Corporate Bonds	5.9%	7.7%
World ex US Bonds	6.1%	10.1%
Russell 1000 Large Cap US Equity	9.6%	15.5%
Russell 2000 Small Cap US Equity	8.8%	19.4%
MSCI EAFE International Equity	7.8%	17.5%
MSCI Emerging Markets Equity	13.9%	23.6%
DJ Wilshire REIT Real Estate	8.5%	13.6%
CRB Futures Index	2.6%	8.8%
Gold, London PM Fix	0.1%	13.0%

*Emerging Equity from January 1988
EM and Ledoit estimated

Case 5: Returns from 2000

Table 5 provides a summary of the historical risk-return estimates from the turn of the century for the expanded set of assets. Very recent data, perhaps fresh in the minds of many investors, may be of interest for comparative purposes.

Table 5: Expanded Assets: Inflation-Adjusted Risk-Return Estimates January 2000 – December 2005

Asset Names	Return	Std dev
US TBills	0.1%	1.3%
Intermediate US Govt Bonds	3.7%	5.1%
Long Term US Govt Bonds	7.3%	10.0%
Long Term US Corporate Bonds	7.2%	8.6%
High Yield US Corporate Bonds	3.8%	8.8%
World ex US Bonds	3.1%	8.9%
Russell 1000 Large Cap US Equity	-1.9%	15.5%
Russell 2000 Small Cap US Equity	5.6%	20.9%
MSCI EAFE International Equity	-0.3%	15.3%
MSCI Emerging Markets Equity	8.1%	21.4%
DJ Wilshire REIT Real Estate	17.7%	14.6%
CRB Futures Index	6.4%	9.9%
Gold, London PM Fix	7.8%	13.4%

** Ledoit estimated

Resampled Efficiency (RE) Optimisation

Markowitz MV Optimisation is generally used to define strategic asset allocations. MV optimised portfolios, however, are well known to be unstable and have poor out-of-sample performance characteristics. Small changes in the inputs often lead to large changes in the optimised portfolios. MV Optimisation is unstable because it implicitly assumes that all risk-return estimates are 100% certain. Investment information is endemically very uncertain and highly inconsistent with the accuracy implied by a digital computer computing a MV Optimisation. As a result MV optimised portfolios are unreliable as a framework for understanding investment value including the role of gold.

RE Optimisation is used to define optimised asset allocations and avoid the limitations of traditional MV Optimisation. RE Optimisation is based on Monte Carlo resampling methods that include uncertainty in risk-return estimates in the definition of portfolio optimality.¹⁹ RE Optimisation is a stable decision making framework and the only provably investment effective portfolio Optimisation technology in the world today (Michaud, 1998, Ch. 6).²⁰

RE Optimisation also allows for estimating the statistical significance of asset allocations.²¹ In Figures 6, 10, and 12, we use RE statistical analysis to determine the significance of gold in the strategic asset allocations on the Resampled Efficient Frontier™ (REF). Statistical significance is determined by examining the weight of gold in the optimal portfolio over all simulated scenarios. We examine the 10th (and 90th) percentile bounds. This is the value that the 10% smallest (largest) values of all simulated gold allocations are less than (greater than). For example, consider the optimal portfolio at the 5% risk level in Figure 6. In 10% of the scenarios, the allocation to gold was 1.0% or less, in another 10% of the scenarios, the allocation was 9.2% or greater. We use this information to be 90% confident that the allocation to gold should be at least 1.0% under the assumptions of Figure 6. Similarly, we can be 80% confident that the appropriate allocation is somewhere between 1.0% and 9.2%.

Case 1 Results

The inflation-adjusted risk-return estimates in Table 1 are roughly comparable to long-term values for many of the assets. Note the large differences in risk and high correlation of gold relative to commodities and the small negative correlation to T-bills but low positive correlations to most other assets.

Figure 4 is a portfolio composition map of the RE optimised asset allocations across the risk spectrum. At very low risk T-bills dominate. At more moderate levels of risk intermediate government bonds are an important asset in the optimal allocation. At higher levels of risk the allocations smoothly increase for long-term government bonds, large cap stocks, small cap stocks, and international equities. In contrast, MV optimised asset allocation would have had a minimal or non-existent role for long-term

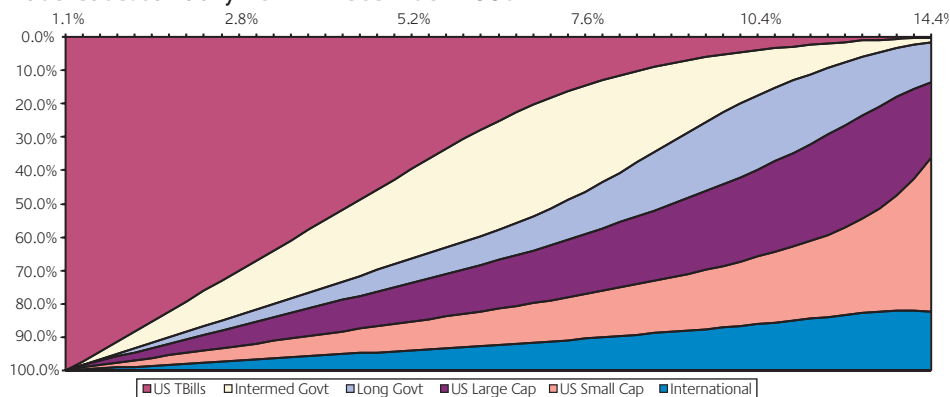
¹⁹ See Michaud and Michaud (2006) for a review and summary of RE Optimisation issues.

²⁰ The computed results are based on 1000 efficient frontier simulations and a Forecast Confidence™ (FC) (patent pending) level of 5. See Michaud and Michaud (2004, 2006).

²¹ See Michaud and Michaud (2002, 2006).

government bonds, small cap, and international equities. The REF provides a more attractive baseline for examining the effect of gold and commodities.²²

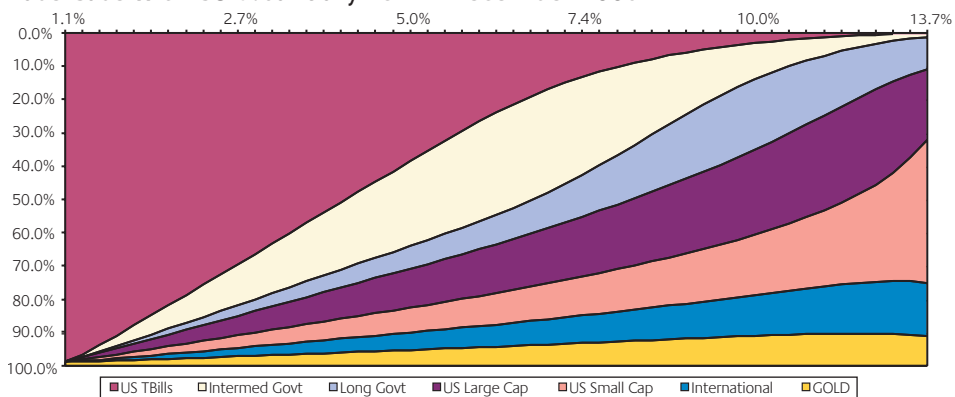
Figure 4: REF Strategic Asset Allocation Composition Map
Base Case: January 1974 - December 2005



*Russell from January 1979. EM estimated.

Figure 5 presents the portfolio composition map of the RE optimised asset allocations including gold. Allocations to gold are substantial, running nearly 10% at higher levels of risk. Importantly, the effect of introducing gold on the other assets is to reduce their allocations in roughly equal measure. In other words, the presence of gold is not a substitute for any particular asset but adds to the definition of portfolio optimality across the risk spectrum.

Figure 5: REF Strategic Asset Allocation Composition Map
Base Case with Gold: January 1974 - December 2005



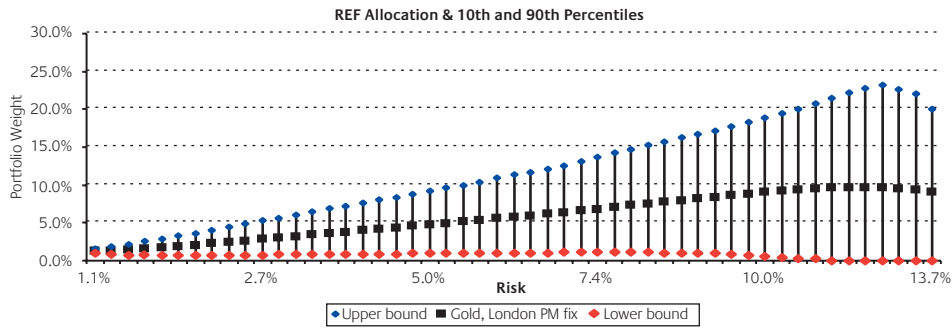
*Russell from January 1979. EM estimated.

Figure 6 provides a statistical analysis of the significance of the gold. The exhibit displays the optimal allocation and the 10th and 90th percentile ranges of gold across the efficient frontier. The 10% left tail does not include zero for all but the most risky

²² Note the very different qualitative results in our REF portfolio composition maps relative to those in Idzorek (2005) particularly for higher risk asset allocations. REF Optimisation always computes more effectively diversified portfolios across the entire risk spectrum and avoids the unjustifiable investment limitations of one hundred percent "optimal" asset allocations in a single asset associated with MV and Feldman (2003) Optimisation procedures.

portfolios. Our evidence indicates that gold is a statistically significant strategic asset at the 10% level of significance for most strategic asset allocations for the last thirty-two years of inflation adjusted historical risk-return data.

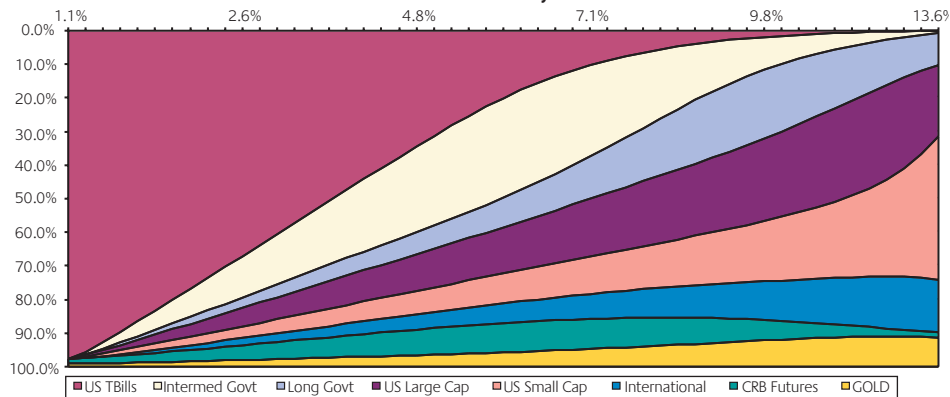
Figure 6: Statistical Significance of Gold REF Allocation and 10th and 90th Percentiles Base Case: 1974-2005



A simple measure of the relative importance of gold in a strategic asset allocation is to compare the optimal allocation to a relatively comparable competing asset. For example, both gold and small cap equities are comparably risky. As Figure 5 shows, except at the highest levels of risk, the allocation to small cap and gold is roughly the same even though the estimate of return is much less. Gold possesses robust diversification properties relative to a similar risky asset.

Figure 7 presents the portfolio composition map of the RE optimised asset allocations including gold and the CRB commodity index. The results show that the commodity futures index is a substantial component of portfolio optimality for low and medium levels of risk but diminishes in importance at high levels of risk. Including gold with commodities does not change the story much. Gold continues to be important in defining portfolio optimality even in the presence of the commodity futures index but is less significant statistically.

Figure 7: REF Strategic Asset Allocation Composition Map Base Case with Gold and CRB Index: January 1974 - December 2005

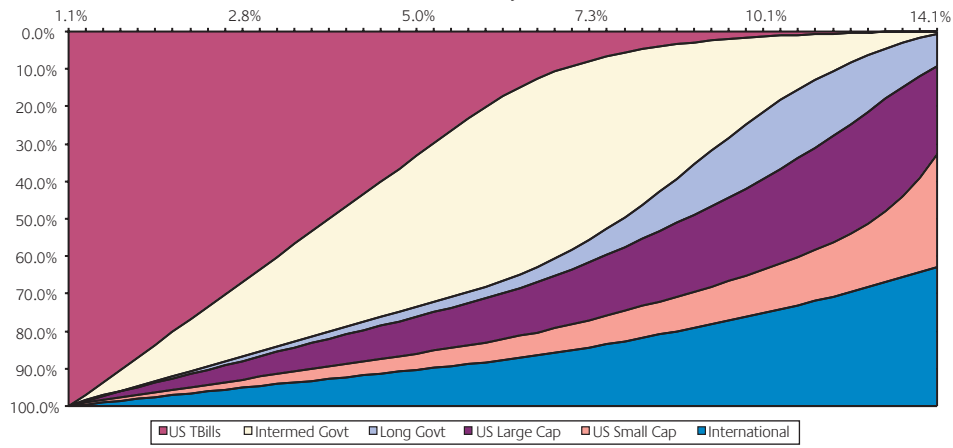


*Russell from January 1979. CRB from January 1982. EM estimated.

Case 2 Results

Figure 8 presents the portfolio composition map of the RE optimised asset allocations for the base case of assets and strategic return premiums in Table 2. It is of interest to compare these results to those from historically estimated risk-returns in Figure 4. The most important differences are the diminished role of long-term government bonds and US small capitalisation and larger role of EAFE.

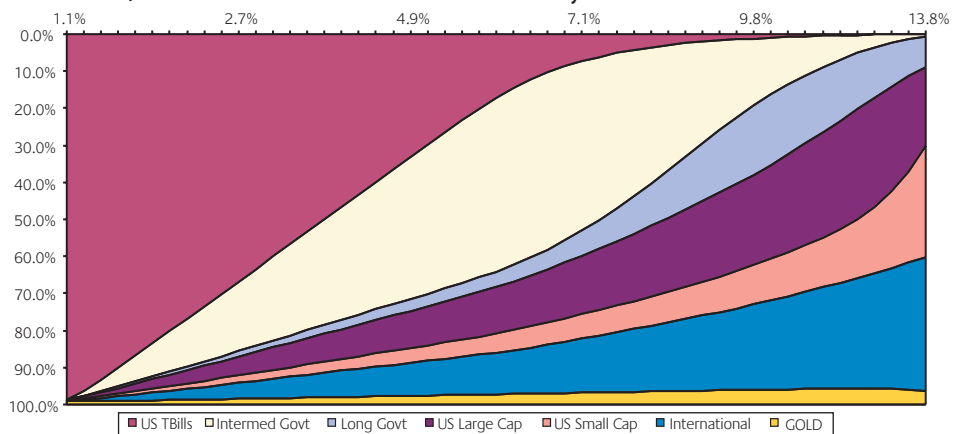
Figure 8: REF Strategic Asset Allocation Composition Map
Base Case with Return Premiums: January 1974 - December 2005



*Russell from January 1979. EM and Ledoit estimated.

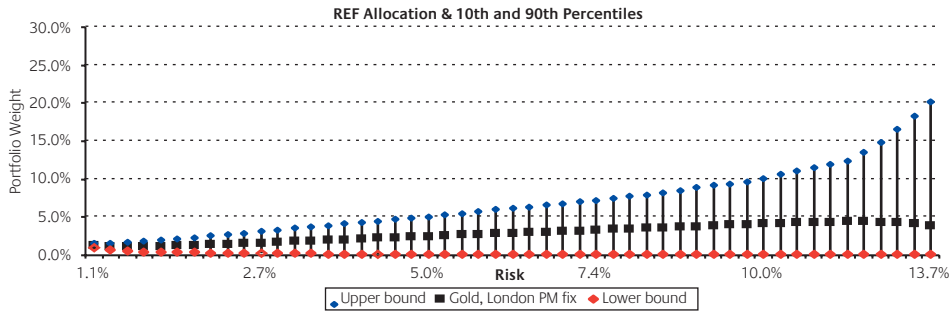
Figure 9 presents the portfolio composition map that includes gold in the previous case. Given a zero real return assumption, gold's smaller allocation relative to Figure 5 is not surprising. Figure 10 examines the extent to which the allocation to gold is statistically significant. The exhibit indicates that gold is statistically significant at the 10% level for roughly the lower half of the risk spectrum. It is of interest to note that gold competes reasonably successfully with small cap as an important alternative diversifying asset at low and moderate risk levels.

Figure 9: REF Strategic Asset Allocation Composition Map
Base Case, Return Premiums and Gold: January 1974 - December 2005



*Russell from January 1979. EM and Ledoit estimated.

Figure 10: Statistical Significance of Gold REF Allocation and 10th and 90th Percentile Base Case, Return Premiums: 1974-2005

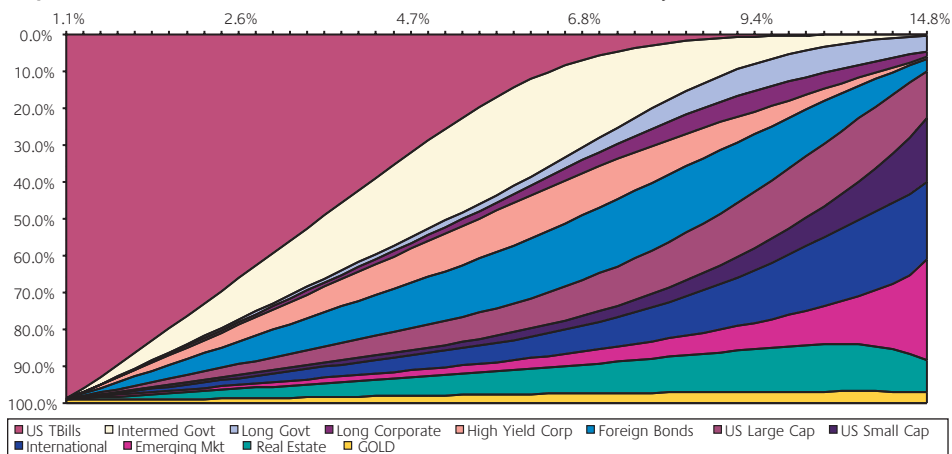


Given the conservative character of our strategic return estimate for gold, a 2 to 4% strategic allocation to gold is investment significant for many large institutional investors. The Optimisation results including commodities are similar as in the previous case and are not reported here.

Case 3 Results

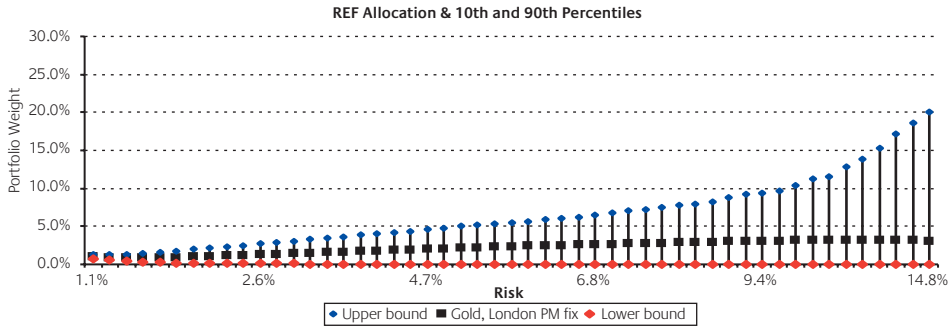
Figure 11 presents the REF portfolio composition map for the expanded set of assets and including gold using the return premiums reported in Table 3. The allocation to gold ranges from 1.1% at low risk to nearly 4% at high risk. Figure 12 provides a 10% significance test for gold and shows that it is statistically significant for roughly the lower half of the risk spectrum of optimal asset allocations. The somewhat smaller allocations relative to the previous case are a reflection of the fact that there are more assets for the optimiser to choose from. The allocations represent a conservative allocation to gold that may be investment significant for large institutional investors. Note that gold competes reasonably well with the equally risky small cap or emerging markets assets as an alternative diversifying asset at low and moderate risk in spite of a much smaller real return assumption.

Figure 11: REF Strategic Asset Allocation Composition Map Expanded Assets, Return Premiums with Gold: January 1974 - December 2005



*Russell from January 1979, CRB from January 1982, High Yield from July 1983, ex-US Bonds from January 1978, Emerging Equity from January 1988, REITS from January 1978. EM and Ledoit estimated

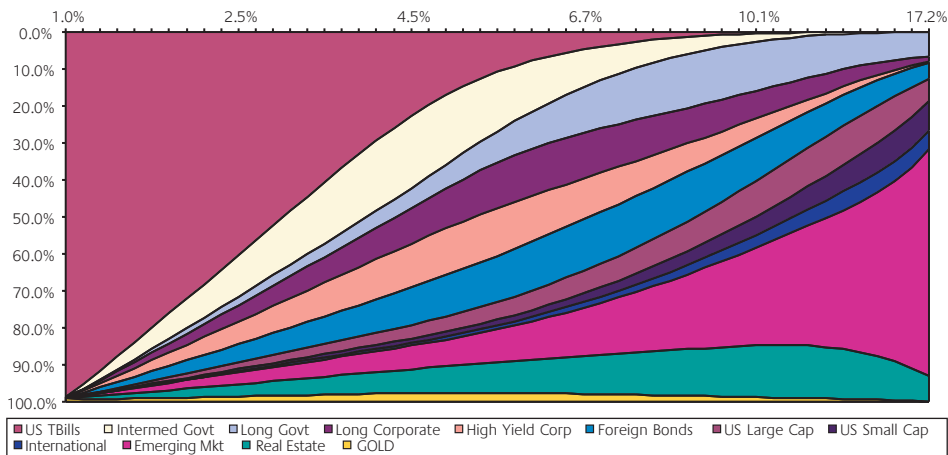
Figure 12: Statistical Significance of Gold REF Allocation and 10th and 90th Percentile Expanded Asset Case, Return Premiums: 1974-2005



Case 4 Results

Figure 13 displays the REF portfolio composition map for the historical risk-return estimates in Table 4 for the expanded set of asset classes and gold. During this period gold was minimally important as part of an optimal portfolio.

Figure 13: REF Strategic Asset Allocation Composition Map Expanded Assets with Gold: January 1986 - December 2005

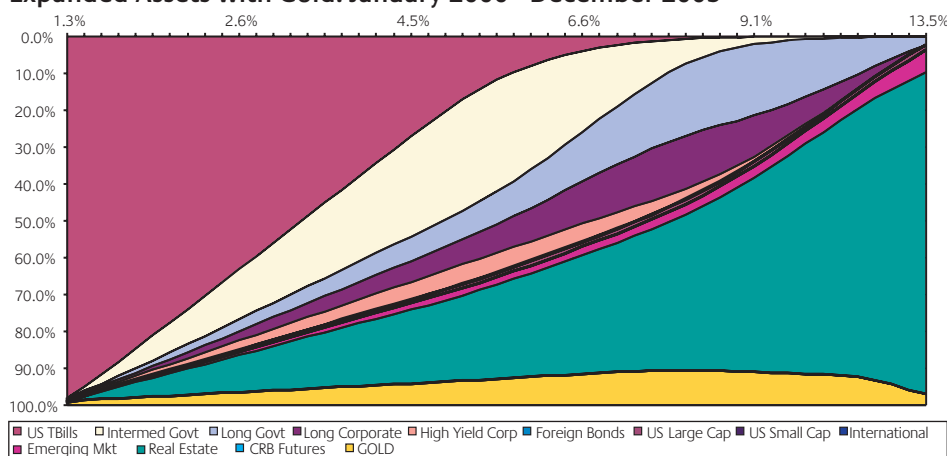


*Emerging Equity from January 1988. EM and Ledoit estimated.

Case 5 Results

Figure 14 displays the REF portfolio composition map for the historical risk-return estimates in Table 5. The results show that the gold allocations ranged from 0.7% at low risk to nearly 8% at middle risk and 2% at high risk. The results for commodities, not reported here, were very significant in this time period, ranging as much as 15% at a peak for middle risk portfolios. Gold would have been a very useful diversifying asset over this period.

**Figure 14: REF Strategic Asset Allocation Composition Map
Expanded Assets with Gold: January 2000 - December 2005**



*EM and Ledoit estimated.

Discussion

favourable results in earlier studies for gold were generally limited by relatively short-term data, not always relevant index comparisons, and unsophisticated and ad hoc statistical estimation, leading to relatively unrealistic return premium assumptions for strategic allocation purposes. In contrast, our empirical risk-return studies found a relatively modest 2% return premium and our strategic return premium cases assumed a conservative zero real return for gold.

Our return series from January 1974 through December 2005 represents roughly all the reliable available data for estimating the investment value of gold for contemporary strategic asset allocation. One problem that arises is that other assets of interest may have missing returns during parts of the period. Analysts have typically used ad hoc and inconsistent methods to deal with incomplete time series assets for risk-return estimation. We employ the EM algorithm to provide a consistent and rigorous methodology for dealing with missing data assets. We also use the Ledoit estimator in all but the first case to improve the reliability of the correlation estimates and Optimisation results.

Commodity future indices are sometimes used as a surrogate for understanding the strategic importance of gold. But a futures contract is not a substitute for a price return index. In addition, many components of commercial commodity futures indices are not relevant to gold. Alternatively, a composite index including gold with other commodities may often have very different risk-return characteristics. For transparency purposes and relevance to our objectives, our study uses a gold only price return index.

The common framework for studying the strategic importance of gold is Markowitz MV efficiency. But MV Optimisation is an unstable and unrealistic framework for understanding portfolio optimality with poor out-of-sample performance characteristics. RE Optimisation is a generalisation of MV Optimisation that includes uncertainty in the definition of portfolio optimality. RE Optimisation provides a more reliable and realistic framework for measuring the importance of assets in an optimised portfolio as well as providing statistical measures to estimate significance.

Conclusions

The objective was to examine the importance of gold in contemporary institutional strategic asset allocation. Our methods avoided the limitations of previous studies by using longer historical periods, conservative estimates of return, a focus on gold price return, improved and consistent risk estimation and enhanced Optimisation technology.

The evidence indicates that gold may be a valuable tactical asset. Gold is highly susceptible to geopolitical factors. During times of relative stability a small positive allocation may be useful. During time periods of abnormally positive economic activity gold returns may reflect multiplier effects associated with cultural issues. During periods of fiscal or monetary mismanagement, crises of various kinds or fundamental changes in the dominant currency, gold may be a very useful asset for hedging risk.

The results also teach that gold may have a comparable portfolio weight to asset classes such as small cap and emerging markets due to its value as a diversifying asset class. With the exception of commodities, gold is not a substitute for other assets but adds diversifying power across much of the risk spectrum. Because they are diversified, commodity indices generally have less short-term volatility than gold, but we find no strategic, as opposed to tactical, reason to believe they have superior return. Gold is a low cost, unbiased, representative commodity. Therefore, given a relatively small portfolio allocation to gold or commodities, the transparency and low correlation of gold with other major asset classes makes it an attractive investment instrument. Depending on the assumptions, empirical evidence indicates that as much as 4% gold allocation may provide useful strategic benefits.

The appropriate allocation to gold is dependent on the risk level of the portfolio. A small allocation to gold has obvious benefits for low risk portfolios due to its low or negative correlation with most other asset classes. Our empirical findings show that a small allocation to gold, in the order of 1 to 2%, is a significant and useful component of low risk portfolios. Gold is also sensible in a balanced portfolio since, as in a CAPM framework, we want to include all sources of economic risk. Our empirical findings show that gold is a statistically significant, though small component of balanced portfolios, in the order of 2 to 4%, depending on assumptions. For high return portfolios it is harder to make a definitive case for gold. Gold clearly is not the asset with the highest long term expected return. However, gold may provide stability in poor markets and economic climates, which can enhance the compound return of aggressive strategic portfolios.

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Appendix

Table 6: Table of pairwise historical correlations for all 13 assets used in the study

Asset Names	Return	Std dev	Correlations														
			US Tbilis	Intrmd US Govt Bonds	Long Term US Corp Bonds	Long Term US Corp Bonds	High Yield US Corp Bonds	World ex US Bonds	Russell 1000 Large Cap US Equity	Russell 2000 Small Cap US Equity	MSCI EAFE International Equity	MSCI Emerging Markets Equity	DJ Wilshire REIT Real Estate	CRB Futures Index	Gold, London PM Fix		
US Tbilis	1.4%	1.1%	1.00														
Intermediate US Govt Bonds	3.6%	6.1%	0.36	1.00													
Long Term US Govt Bonds	4.9%	10.9%	0.30	0.90	1.00												
Long Term US Corporate Bonds	4.8%	9.7%	0.31	0.87	0.94	1.00											
High Yield US Corporate Bonds	6.4%	7.5%	0.22	0.24	0.30	0.43	1.00										
World ex US Bonds	5.9%	9.6%	0.22	0.51	0.45	0.45	0.05	1.00									
High Yield US Corporate Bonds	6.4%	7.5%	0.22	0.24	0.30	0.43	1.00										
World ex US Bonds	5.9%	9.6%	0.22	0.51	0.45	0.45	0.05	1.00									
Russell 1000 Large Cap US Equity	9.8%	15.4%	0.16	0.19	0.24	0.28	0.52	0.06	1.00								
Russell 2000 Small Cap US Equity	10.4%	19.6%	0.11	0.09	0.13	0.19	0.57	0.00	0.84	1.00							
MSCI EAFE International Equity	7.2%	17.0%	0.18	0.16	0.17	0.21	0.36	0.43	0.58	0.52	1.00						
MSCI Emerging Markets Equity	13.1%	23.1%	0.06	-0.11	-0.10	0.02	0.45	0.00	0.60	0.63	0.58	1.00					
DJ Wilshire REIT Real Estate	10.5%	14.9%	0.11	0.20	0.21	0.26	0.43	0.09	0.53	0.63	0.32	0.31	1.00				
CRB Futures Index	1.9%	9.0%	-0.02	-0.11	-0.10	-0.09	0.00	0.09	0.09	0.12	0.22	0.21	0.10	1.00			
Gold, London PM Fix	2.1%	19.7%	-0.17	0.01	0.00	0.00	-0.06	0.14	0.05	0.12	0.20	0.08	0.08	0.45	1.00		

* Russell from January 1979, CRB from January 1982, High Yield from July 1983, ex-US Bonds from January 1978, Emerging Equity from January 1988, REITs from January 1978. Results calculated from inflation-adjusted historical data using longest available historical period.

Notes

Research Studies available on request

No. 32 *Short-run and Long-run Determinants of the Price of Gold* Eric J. Levin & Robert E. Wright, June 2006

No. 31 *Gold in Central Bank Reserves* Gordon Gemmill, January 2006

No. 30 *Gold as a Hedge Against the US Dollar* Forest Capie, Terence C. Mills, Geoffrey Wood, September 2004

No. 29 *Should Gold Exporters Peg Their Currencies to Gold?* Jeffrey Frankel, November 2002

No. 28 *America's Deficit, the Dollar & Gold* Tim Congdon, November 2002

No. 27 *Deregulation of Gold in India* Himadri Battacharya, November 2002

No. 26 *The IMF and Gold (Revised Edition)* Dick Ware, May 2001

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