



Estimation Error and the “Fundamental Law of Active Management”

By

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Abstract

According to widely referenced applications of the Grinold (1989) “Fundamental Law” theory, simply adding more securities to an optimization universe, adding more factors to a forecast return model, trading more frequently, or reducing more constraints can add investment value to an optimized investment strategy. We show with intuitive discussion followed by a novel simulation study that applications of the Grinold theory for optimized portfolio design are often unreliable and self-defeating. Critical limitations are due to ignoring estimation error (Michaud 1989) and constraints required in practical applications. A substantial fraction of professional actively managed funds may be negatively impacted.

Keywords: active management, portfolio construction, asset allocation, quantitative finance, Fundamental Law of Active Management, optimization

Benchmarks arise naturally in judging asset manager competence and for meeting investment goals. An active investment manager typically claims to provide enhanced return on average relative to a given benchmark or index for a given level of residual risk. The information ratio (IR) – estimated return relative to benchmark per unit of residual risk or tracking error – is a convenient and ubiquitous framework for measuring the value of active investment strategies.

The Grinold (1989) “Fundamental Law of Active Management” asserts that the maximum attainable IR is approximately the product of the Information Coefficient (IC) times the square root of the breadth (BR) of the strategy.* The IC represents the manager’s estimated correlation of forecast with ex post residual return while the BR represents the number of independent bets or factors associated with the strategy. GK assert that the “law” provides a simple framework for enhancing active investment strategies. While a manager may have a relatively small amount of information or IC for a given strategy, performance can be enhanced by increasing BR or the number of independent bets in the strategy. In particular, they state “The message is clear: It takes only a modest amount of skill to win as long as that skill is deployed frequently and across a large number of stocks.”† Their recommendations include increasing trading frequency, size of the optimization universe, and factors to models for forecasting return. Assumptions include independent sources of information and IC constant for each added bet or increase in BR.

Clarke, de Silva and Thorley (2002, 2006) (CST) generalize the Grinold formula by introducing the “transfer coefficient” (TC). TC is a scaling factor that measures how information in individual securities is “transferred” into optimized portfolios. TC represents a measure of the reduction in investment value from optimization constraints. This widely influential article has been used to promote many variations of hedge fund, long short, alternative, and unconstrained investment strategies.‡

A significant literature exists on applying Grinold theory and variations for rationalizing various active equity management strategies. Extensions include Buckle (2004), Qian and Hua (2004), Zhou (2008), Gorman et al (2010), Ding (2010), Huiz and Derwall (2011), Ding and Martin (2017). Industry tutorials and perspectives include Kahn (1997), Kroll et al (2005), Utermann (2013), Darnell and Ferguson (2014), Menchero (2017). Teachings include the Chartered Financial Analyst (CFA) Institute Level 2, the Chartered Alternative Investment Analyst (CAIA) Level 1 and many conferences and academic courses in finance. Texts discussing the formula and applications include Focardi and Fabozzi (2004), Jacobs and Levy (2008), Diderich (2009), Anson et al (2012),

* The Grinold formula is analytically derived and based on an inequality-unconstrained maximization of quadratic utility. It should not be confused with Markowitz (1952, 1959) which assumes linear (inequality and equality) constrained portfolios and requires quadratic programming techniques to compute the MV efficient frontier. In particular, the Markowitz efficient frontier is generally a concave curve in a total or residual return framework while in Grinold (see e.g., GK 1995, p. 94) it is a straight line emanating from a zero residual risk and return benchmark portfolio. The Grinold derivation also assumes IC small, in the order of 0.1.

† GK (1995, Ch. 6, p. 130), also GK (1999, Ch. 6, p. 162).

‡ One example is Kroll et al (2005). Michaud (1993) was the first to note possible limitations of the long-short active equity optimization framework.

Schulmerich et al (2015). A very substantial fraction of globally professionally managed funds is estimated to employ optimized portfolio design principles that are applications of Grinold theory.

We show with both qualitative discussion and a novel Monte Carlo simulation study that the GK and CST proposals based on Grinold theory for optimized portfolio design are unreliable and often self-defeating. Applications are based on a theory that assumes no estimation error in plays of the investment game as in roulette in a casino.* Unlike roulette, the investment game signal is not constant and may often be negative. As we show in Figure 1 in our simulation study, there is an enormous difference how optimized strategies perform on average out-of-sample when estimation error is assumed (Michaud 1989). Our simulation results generalize the classic JK and FS simulation studies and rationalize the empirical “1/N” results as in deMiguel (2007). The GK and CST precepts of active management and a cohort of published papers have been largely unchallenged for more than twenty-five years. The limitations we identify are estimated to affect a substantial fraction of globally professionally managed active equity funds.

The outline of the paper is as follows. Section 1 presents the Grinold formula, the GK and CST prescriptions for active management with reference to the GK casino management rationale. Section 2 discusses the limitations of the GK and CST prescriptions from an intuitive investment perspective. Section 3 provides a discussion of properties of index-relative mean-variance (MV) optimization and previous simulation studies relevant to our results. Section 4 presents our Monte Carlo simulation study that demonstrates that applications associated with the fundamental law are likely invalid and often self-defeating. Section 5 provides a summary and conclusions.

1.0 Grinold’s Fundamental Law of Active Management

Grinold (1989) theory is an approximate decomposition of the information ratio (IR) generally associated with active equity optimized portfolio management. Grinold shows that the MV optimization of an inequality unconstrained residual return investment strategy is approximately proportional to the product of the square root of the breadth (BR) and the information correlation (IC).† Mathematically,

$$IR \cong IC * \sqrt{BR}$$

where IR = information ratio = (alpha) / (residual or active risk)
 IC = information correlation (ex ante, ex post return correlation)
 BR = breadth or number of independent sources of information.

The formula teaches that successful active management depends on both the information level of the forecasts and the breadth associated with the optimization strategy. However, GK and

* To be clear, we note that the term “estimation error” in this manuscript refers to Monte Carlo simulation experiments that measure how estimates of optimization parameters impact out-of-sample investment performance, and not, as in Zhou (2008) or Kritzman (2010), to refer to statistical estimation issues.

† The detailed derivation is given in GK, Ch. 6, and Technical Appendix.

CST go further. They apply the Grinold formula to assert that investment performance can be enhanced with only a modest amount of information (IC) by increasing the number of assets in the optimization universe, the number of forecast factors, more frequent trading and reducing optimization constraints.

GK use a casino roulette game to rationalize applications of the Grinold formula to asset management in practice.* The probability or IC of a winning play (for the casino) of the roulette game is small but more plays (breadth) lead to the likelihood of more wealth. However, there are important differences between the play of a roulette game in a casino and the play of an investment game in practice. In the casino context, probability of a winning play or IC is known, positive, and constant. In an investment game, the IC is unstable and may often be negatively related to return. In the context of estimation error, increasing the number of plays of an investment game may often be undesirable. While interesting, the casino game rationale for rationalizing applications of the Grinold formula to actual investment practice is invalid.

2.0 Discussion of GK and CST Prescriptions

GK and CST propose four principles of optimized portfolio design for enhanced investment value in an index-relative MV optimization framework. We discuss the limitations of each prescription in turn from an intuitive point of view.

2.1 Large Optimization Universe Fallacy

GK argue that investment value increases with the size of the optimization universe conditional that the IC is roughly equal for all securities in the optimization universe. How realistic is this assumption?

For a small universe of securities, the assumption of uniform average IC may be tenable. Small universes may be homogeneous in character. However, for a large and expanding optimization universe, it seems untenable to assume uniform average IC across all subsets. Any manager will naturally use the securities with the best information first. While, theoretically, adding more assets may add marginally to breadth, all other things the same, it is also likely to result in less predictable securities and reduce the overall average IC level of the universe. A lower average IC may cancel any gains made from increasing breadth.

The issue can be framed in a more common practical setting. Consider an analyst suddenly asked to cover twice as many stocks. Given limitations of time and resources, it is highly unlikely that the analyst's average IC is the same for the expanded set of stocks. Issues of resources and time are the primary reasons why analysts tend to specialize in areas of the market or use managers in investment strategies that limit the number of securities that they cover. In practice, many traditional managers limit the number of securities they include in their active portfolio to not much more than twenty or fifty. Except for relatively small asset universes, the average IC and

* The casino roulette game framework is very consistent with the assumptions used in the Grinold derivation in GK (1995, 1999, Ch. 6. App.)

overall level of IR may often be a decreasing function of the number of stocks in the optimization universe, all other things the same. GK seem to be aware of these limitations, for example as suggested by their statement “The fundamental law says that more breadth is better, provided the skill can be maintained.” Nevertheless, average IC and optimization universe size are likely negatively correlated in applications.

2.2 Multiple Factor Model Fallacy

Large stock universe optimizations typically use indices such as the S&P500, Russell 1000 or even a global stock index as benchmarks. In this case, individual analysis of each stock is generally infeasible and analysts typically rely on factor valuation frameworks for forecasting alpha. For example, stock rankings or valuations may be based in part on an earnings yield factor.* As GK note, if earnings yield is the only factor for ranking stocks, there is only one independent source of information and breadth equals one.

In the Grinold formula, the IR increases with the number of independent positive significant factors in the multiple valuation forecast model. However, in practice, asset valuation factors are often highly correlated and may often be statistically insignificant, providing dubious out-of-sample forecast value.† Finding factors that are reasonably uncorrelated and significantly positive relative to ex post return is no simple task.

Factors are often chosen from a small number of categories considered relatively uncorrelated and positively related to return such as value, momentum, quality, dividends, and discounted cash flow.‡ The breadth of multiple valuation models may often be very limited independent of the size of the optimization universe.§,** As in adding stocks to an optimization universe, adding factors at some point may include increasingly unreliable factors that are likely to reduce, not increase, the average IC of an investment strategy.

Michaud (1990) provides a simple illustration of the impact of adding factors to a multiple valuation model. While adding investment significant factors related to return can be additive to IC, it can also be detrimental in practice. There is no free lunch. Adding factors can as easily reduce as well as enhance investment value, and the number of factors that can be added while maintaining a desirable total IC is generally limited in practice.

* Some standard methods for converting rankings to a ratio scale to input to a portfolio optimizer include Farrell (1983) and references.

† There is a practical limit to the number of independent investment significant factors even in many commercial risk models, often far less than ten.

‡ Standard methods such as principal component analysis for finding orthogonal risk factors are seldom also reliably related to return over independent periods.

§ See e.g., Michaud (1999).

** While principal component or factor analysis procedures for identifying orthogonal factors in a data set may be used, most studies find no more than five to ten investment significant identifiable factors that are also useful for investment practice.

2.3 Invest Often Fallacy

GK recommend increasing trading period frequency or “plays” of the investment game to increase the BR, and thus the IR of a MV optimized portfolio. The Grinold formula assumes trading decision period independence and constant IC level. However, almost all investment strategies have natural limits on trading frequency.* For example, an asset manager trading on book or earnings to price will have significant limitations increasing trading frequency smaller than a month or quarter. Reducing the trading period below some limit will generally reduce effectiveness while increasing trading costs.

Fundamentally, trading frequency is limited by constraints on the investment process relative to investment style.† Deep value managers may often be reluctant to trade much more than once a year while growth stock managers may want to trade multiple times in a given year. Increased trading, to be effective, requires increasing the independence of the trading decision while maintaining the same level of skill. This will generally require increased resources, if feasible, all other things the same. The normal trading decision period should be sufficiently frequent, but not more so, in order to extract relatively independent reliable information for a given investment strategy and market conditions.

It is worth noting that the notion of normal trading period for an investment strategy does not imply strict calendar trading. Portfolio drift and market volatility relative to new optimal may require trading earlier or later than an investment strategy “normal” period. In addition, a manager may need to consider trading whenever new information is available or client objectives have changed. Portfolio monitoring relative to a normal trading period including estimation error is further discussed in Michaud et al (2012).

2.4 Remove Constraints Fallacy

Markowitz’s (1952, 1959) MV optimization can accommodate linear equality and inequality constraints. In actual investment practice, MV optimized portfolios typically include many linear constraints. This is because MV optimized portfolios are sensitive to estimation error in estimated inputs that often lead to unintuitive and impractical portfolios (Michaud 1989). Constraints are often imposed to manage instability, ambiguity, poor diversification characteristics, and enhanced out-of-sample performance. However, constraints added solely for marketing or cosmetic purposes may result in little, if any, investment value and may obstruct the deployment of useful information in risk-return estimates.

In general, inequality constraints are necessary in practice. Inequality constraints reflect the financial fact that even the largest financial institutions have economic shorting and leveraging limitations. Recently, Markowitz (2005) demonstrates the importance of practical linear

* Special cases may include proprietary trading desk strategies where the information level is maintained at a reasonable level and trading costs are nearly non-existent. Other cases, such as high frequency and algorithmic trading are arguably not investment strategies but very low-level IC trading pattern recognition relative to highly sophisticated automated liquidity exchange intermediation.

† Trading costs and market volatility are additional considerations.

inequality constraints in defining portfolio optimality for theoretical finance and the validity of many tools of practical investment management. Long-only constraints limit liability risk, a largely unmeasured factor in many risk models and often an institutional requirement. Regulatory considerations may often mandate the use of no-shorting inequality constraints. Performance benchmarks may often mandate index related sets of constraints for controlling and monitoring investment objectives. Moreover, inequality constraints limit the often-negative impact of estimation error in out-of-sample performance (Frost and Savarino 1988).

3.0 Testing GK and CST proposals

Investment managers often use a back test to demonstrate the likely value of a proposed investment strategy. In this procedure, a factor or strategy is evaluated on how it performed for historical data over some period. While the benefit of a back test may be practicality, no reliable prospective information is possible by definition. It is no less, and no more, than what happened over some historical period. Back tests are notorious for misleading investors, resulting in loss of wealth, careers, and dissolution of firms. Investors should be keenly aware of the serious limitations of any back test as evidence of the reliability of any factor relationship or investment strategy.*

A simulation study is a far more reliable framework for testing the value of optimized investment strategies. Such a procedure evaluates the likely out-of-sample performance of an in-sample optimized portfolio for many realistic investment scenarios.

In the following sections, we explain the summary statistics used to evaluate the out-of-sample performance of investments from following the prescriptions of the fundamental law, describe the simulation test framework in detail, and discuss the results of our simulation experiment.

3.1 Portfolio simulation study framework

Our study uses a framework similar to other well-known simulation studies for portfolio construction methods.[†] In this framework, a referee is assumed to know the true means, standard deviations, and correlations for a set of assets and consequently the true max Sharpe ratio (MSR) for an optimized portfolio of those assets. The players do not know the referee's true MV parameters. The players receive simulated returns based on the referee's parameters, so they can only observe the truth obscured by estimation error, as is true for all real-world investment managers. The players then compute optimal weights for their strategies and report the simulated MSR optimal portfolios for the referee to score. The referee determines the true Sharpe Ratios (SRs) for the simulated MSR optimal portfolios. The procedure is repeated many times for referee-simulated returns, and averages of true simulated optimal portfolio SRs computed for each player. In this way, the out-of-sample performance of each player's strategy can be compared, and the better strategy determined.

* Even long-term academic studies remain susceptible to unreliability in practice.

[†] For example, Jobson and Korkie (1981).

3.2 Prior MV optimization simulation studies

Jobson and Korkie (JK) (1981) provide the classic study of the effect of estimation error on the out-of-sample investment value of unconstrained MV optimized portfolios.* In their study, the referee's truth is based on historically estimated MV inputs for twenty stocks. They Monte Carlo simulate returns assuming a multivariate normal distribution of five years of monthly return data. They find that the average of the true SRs, as measured by the referee, of simulated MSR optimal portfolios, was twenty-five percent of the true MSR of the referee's optimal portfolio. In addition, they show that equal weighting substantially outperforms the optimized portfolios.† They conclude that unconstrained MV optimization is not recommendable for practice.

Frost and Savarino (FS) (1988) perform a related simulation study for long-only MV optimized portfolios. They find that additional constraints may often add investment value to the out-of-sample performance of MV optimized portfolios. Economically realistic constraints may often act like Bayesian priors focused on portfolio structure enforcing rules representing legitimate information not contained in the optimization inputs. Such restrictions can mitigate estimation error in risk-return estimates implicitly by forcing the simulations towards more likely optimal portfolios.

We note that the JK and FS studies contradict the theoretical results of CST for two different size stock universes. Our study confirms and generalizes their results conditional on optimization universe size.

4.0 Simulating Adding Breadth while Maintaining Information Levels

In the standard interpretation of the Grinold formula, each spin of the roulette wheel adds one unit of breadth to the investment game. There is no uncertainty as to whether additional spins will continue to be advantageous for the house even when the odds are only slightly in its favor. In our simulations, the critical deviation from the GK roulette wheel framework is that estimation error of the probability of a win for the investment house is neither known nor constant. Our objective is to construct a Monte Carlo simulation in the context of estimation error where each randomly selected incremental asset has variable information but a constant expectation of adding one unit of breadth. Therefore, the simulated optimized portfolios will be affected by estimation error, but the average of the simulations will exhibit constant incremental breadth.

4.1 Simulation Methodology

We begin with a sample of historical market return data‡ of 500 stocks that will be the basis for our simulations. This particular dataset is immaterial to our argument. What is essential is that

* A budget (sum to one) constraint is also assumed. Note that the JK study applies equivalently to unconstrained quadratic utility portfolio optimization, a framework widely used in financial theory and for the development of many investment strategies.

† An equal weighted portfolio is a simple way to compare the optimality of unconstrained optimized portfolios.

‡ We use a recent history of US market data (1994-2013) of publically available data to create our master asset list and corresponding mean and variance parameters. We selected all the assets from the largest 1000 in market capitalization with contiguous data from the period, excluding returns greater than 50% or less than -50% per month. We were able to find 544 stocks that met our criteria. Parallel experiments with shorter histories were also run to

the master dataset represents a realistic set of expected returns and full-rank covariance matrix for the largest sample size of the experiment.*

We propose a novel simulation framework that consists of random sampling without replacement of increasing size subsets of the 500 stocks of the referee's risk-return estimates from the master optimization universe. The averaging of the results of thousands of samplings without replacement from the given 500 stock universe with increasing size provides a solution to estimating the Grinold theoretical concept of increasing a unit of breadth for increasing number of assets. In this way, the functional form of average out-of-sample simulation performance can be compared to Grinold theory prediction of a monotonic increasing concave function of breadth.

We Monte Carlo simulate returns assuming a multivariate normal distribution for the referee's mean and covariance matrix. Each simulation consists of sampling without replacement of the 500 stocks of increasing size to 500 assets. The referee's truth is computed by independently adding assets to the referee's expected return and covariance matrix. The problem of computing a sample covariance from simulated returns is avoided by assuming the referee's covariance. This assumption eliminates non-full-rank covariance estimation from simulated returns as a plausible explanation of our results.[†] It also means that our results represent a very generous upper bound of average out-of-sample performance for actual practice.

4.2 Unconstrained, Long-only, and Equal Weight

Figure 1 reports the results of our simulation studies. It consists of three panels of simulation results for 0.1, 0.2, and 0.3 IC levels of estimation error from optimization universes from five to 500 assets. Each value presented on the graph is averaged from 16,000 samplings without replacement optimizations. The three graphed series in each panel show the out-of-sample average SRs resulting from three different optimization methods. The "unconstrained" series displays the out-of-sample averages of the true SRs for the simulated unconstrained MSR portfolios, the "equal weight" series displays the average true SRs of equal weighted portfolios, and the "constrained" series reflects the out-of-sample averages of the true SRs of simulated long-only MSR portfolios. The fourth graph in green reflects the average SRs for unconstrained

investigate if selection bias affects results, with no positive findings, so we present the twenty-year history here. Readers wishing to replicate our experiment can access our data at http://newfrontieradvisors.com/media/1657/estimation_error_and_the_fundamental_law_data.csv.

* A principal components decomposition of our referee's covariance matrix confirms that none of the independent dimensions of the system vanish. All of the eigenvectors are needed to replicate our forecast to reasonable precision. If some of the eigenvalues were vanishingly small, the practical answer to the question of breadth would be quite different from the mathematically rigorous one. However, the full covariance matrix of 500 assets in our dataset has a smallest eigenvalue of over 10 basis points, which is likely significant for most definitions of statistical significance. This would correspond to an annualized standard deviation of approximately 11%, which is substantial by most measures. The submatrices of smaller portfolios tend to have even greater values for the smallest eigenvalue. This line of reasoning confirms that the effective breadth of a sample of size N from our universe is identically N in a practical sense as well as the theoretical one.

[†] In particular, this assumption avoids the issues in Fan et al (2008).

MV optimization for the no estimation error case as a function of optimization universe size in Grinold theory.

Our simulations confirm and generalize the simulation study results in JK and FS and provide a rationale for the empirical results in deMiguel et al (2007). In particular, equal weight is far superior to unconstrained optimization, as in JK, for optimization universes of modest size, such as in asset allocation studies and consistent with the larger optimization universe in FS, where long only dominates unconstrained and equal weight. Our results provide a single consistent framework for summarizing and extending the classic results in historical studies on estimation error relative to the Grinold framework.

The three levels of the Grinold assumed IC: 0.10, 0.20, and 0.30 is computed by varying the number of periods of simulated returns for each size universe. Because of the Monte Carlo nature of our experiment, the average realized ICs for each sample size are close but not exactly equal to target.* The observation sizes for each target IC were determined by calibrating the largest portfolio size (500) for the experiment. While IC levels greater than 0.10 are not formally applicable to predictions from the Grinold formula, our simulations transcend assumptions in the theory and may have important teachings in other investment applications. While each of the reports for stock subsets without replacement will necessarily reflect randomness of adding stocks, averaging over 16,000 such simulations should represent a very reliable estimate of additive breadth for a realistic data set of historical returns. Our results should be reasonably representative for similar datasets of practical interest.

In the case of IC equal to 0.30, the out-of-sample unconstrained performance nearly attains the level of the constrained case for the largest sample size of 500 assets. However, these experiments avoid any consideration of financial frictions or costs that would limit the investment value of large universe optimized portfolios.† Our assumption of an error free covariance matrix further upward biases our simulations. Our results should provide convincing evidence of the limitations of ignoring estimation error for portfolio optimization design.

4.3 Further Discussion

Our deliberate optimism on how additive breadth is modeled when increasing the size of the optimization universe in the simulations has important implications. All of the assets in the simulation universe are assumed to have some investment value. Consequently, an investor is little harmed by putting portfolio weight on a “wrong” asset. In the real world, constraints often limit the harm caused by misinformation. In a truly chaotic world with a lot of estimation error and bias, the equal weighted portfolio, which uses no “wrong” information to distinguish among assets, can be hard to beat, for small optimization universes such as in asset allocation strategies.

The consistent slow rising level of unconstrained simulated optimized portfolio average true SRs as universe size increases is a necessary artifact of our simulation framework. This is because, by

* Results are available on request.

† See for example Grossman and Stiglitz (1988).

design, our simulations assume a consistent level on average of realized IC regardless of universe size. In practice, many investment strategies have an optimal universe size. Beyond some point, increasing universe size is likely to be self-defeating in practice.

5.0 Summary and conclusions

Our narrative does not contradict the simple intuition that investment performance is a function of skill and breadth. It is always true that it is better to have more reliable information (IC) and more additional investment opportunities to apply it (BR). The crucial limitation associated with applications of the theoretical Grinold square root law result from a failure to consider the impact of estimation error on MV utility portfolio optimization in practical applications. In addition, Grinold's formula is based on the false premise that IC and BR can be thought of independently. However, even under the highly idealized conditions of our simulation study where BR can be additively applied while holding IC level constant, the results in Figure 1 dramatically contradict the predictions of Grinold theory.

The results in Figure 1 generalize and extend the classic JK and FS simulation studies and help rationalize the empirical "1/N" results in DeMiguel et al (2007). Our simulation framework enables measurement of the theoretical notion of Grinold additive "breadth."

Our results have important implications for contemporary investment practice. For more than twenty-five years, Grinold theory applications have often been considered the bible of professionally managed quantitative equity funds. Our results indicate that the popular four principles of optimized portfolio design proposed by GK and CST from applications of the Grinold formula – frequent trading, adding securities, adding forecast factors, and removing constraints – are not reliably beneficial and may often be self-defeating. Estimation error affects the reliability of many of the results in the cohort of research related to applications of Grinold theory. Many rationales for investing in hedge funds, long short, alternatives, and unconstrained strategies may be impacted.

The necessary conditions for reliably winning the investment game remain the fundamental principles of reliable long-term asset management: 1) investment significant information and high quality investible assets relevant to a given size optimization universe; 2) economically meaningful constraints; and 3) properly implemented estimation error sensitive portfolio optimization technology.

The ultimate root of the failure of applications of the theoretical Grinold formula is one of many examples of the fundamental and ubiquitous fallacy in many areas of social science of regarding inference from in-sample statistics and fixed probability models as the full measure of uncertainty.*

* Knight (1921) and Weisberg (2014).

Figure 1
Average Maximum Sharpe Ratios by Information Level, Referee Covariance

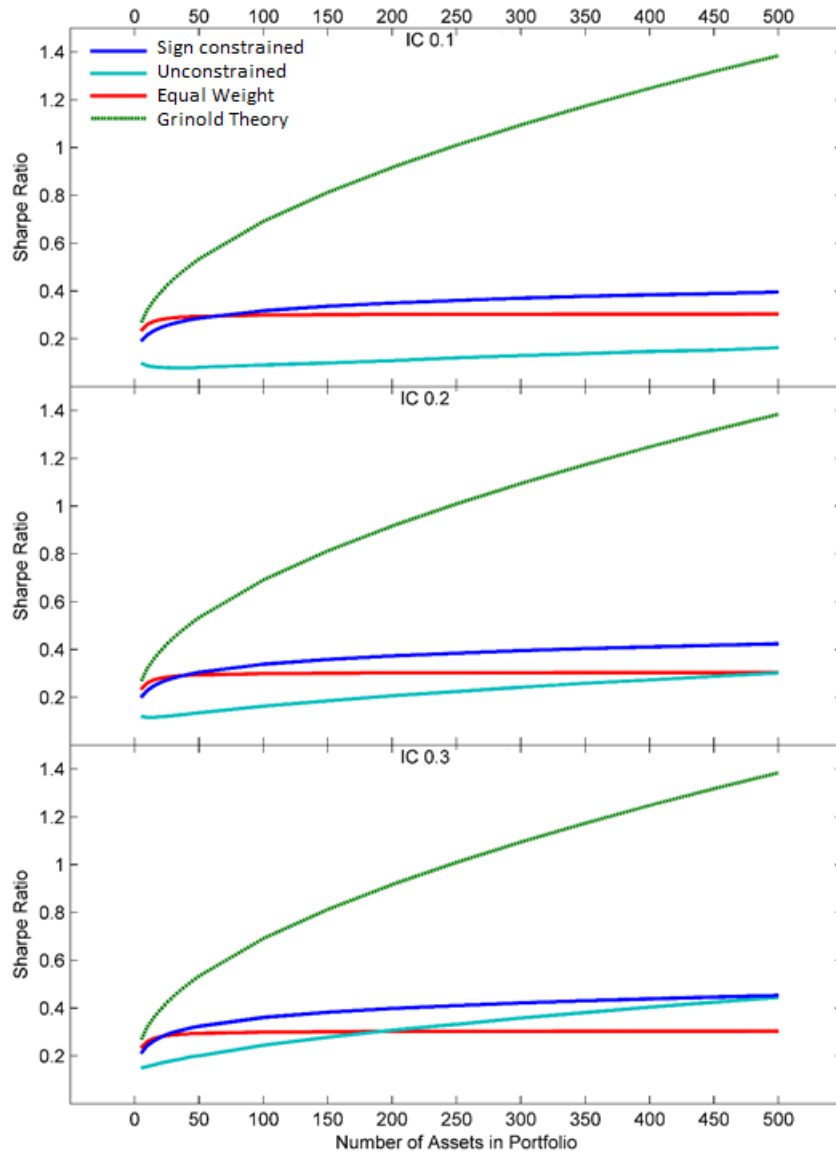


Figure 1: Average SRs for three different portfolio construction methods and three different information coefficients for the equity optimization case, using the referee's covariance matrix. Target information coefficients are not precisely attained by the simulations and realized ICs are shown in Table 1. This experiment was run on many simulations of up to 500 U. S. stocks which had at least 20 years of contiguous monthly price data ending in December 2013.

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