Morningstar® vs. Michaud Optimization

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Classical linear constrained Markowitz (1952, 1959) mean-variance (MV) optimization has been the standard for defining portfolio optimality for more than fifty years. However, Markowitz efficient portfolios are known in practical application to be unstable and highly sensitive to estimation error in risk-return inputs. Michaud optimization (1998, 2008a, 2008b) is a U.S. patented generalization of linear constrained Markowitz MV efficiency that uses modern statistical resampling technology to address estimation error and instability in portfolio optimization.[1] The Morningstar® Encorr® software also features a portfolio optimization procedure that uses the terms "resampling" and "resampled frontiers." In this report we discuss the similarities and differences of the two methods and illustrate the results using identical inputs and portfolio optimality criteria. We show that the procedures are fundamentally different and the results typically very dissimilar. While the Michaud portfolios are investment intuitive, stable, and well diversified across the entire efficient frontier the Morningstar portfolios are often inconsistent with sensible perceptions of diversification and generally reflect serious limitations as alternatives to MV optimization limitations. The lack of theoretical framework for the procedure and the non-uniqueness of the solutions defeats Morningstar claims of superior investment value relative to Markowitz or Michaud optimality.

The plan of the paper is as follows. Section 1 describes the resampling of risk-return estimates implicit in the Morningstar and Michaud procedures. Section 2 describes the different efficient frontier averaging process used in Morningstar and Michaud optimization and illustrates the efficient frontiers with a twenty asset historical return data set. Section 3 provides composition map analyses of the portfolios of the efficient frontiers produced by the three optimization procedures. Section 4 summarizes and concludes.

1.0 The Resampling Process

Both the Michaud and Morningstar optimizers are based on resampling methods originally described in Michaud (1998). Monte Carlo techniques are used to simulate alternative risk-return estimates that generate alternative statistically equivalently optimal Markowitz MV efficient frontiers. Resampling the inputs is the method of choice for understanding uncertainty endemic in investment information.

Figure 1 illustrates the resampling process of the simulated Markowitz MV efficient frontiers for the data taken from Michaud (2008b). The data set consists of twenty U.S. stocks randomly chosen from 100 largest capitalization stocks in the S&P 500 index with continuous monthly returns from January 1997

[1] Resampled Efficient optimization or Michaud optimization was invented and patented by Richard Michaud and Robert Michaud, U.S. patent 6,003,018. Worldwide patents pending. New Frontier Advisors is the exclusive worldwide licensee.

through December 2006. The list of stocks, their annualized average returns, standard deviations and correlations over the period and further details are given in Appendix B.

The red curve displays the Markowitz sign-constrained MV efficient frontier for the data. The cyan curves are each Markowitz sign-constrained MV efficient frontiers for resamplings of the risk-return estimates. The display in Figure 1 is limited to twenty-five simulated alternative resampled MV efficient frontiers for pedagogical purposes. In practice, thousands of resampled MV efficient frontiers may be computed.

Figure 1 shows that simulated MV efficient frontiers may have much less or much more estimated risk and/or return than the original Markowitz efficient frontier. The set of simulated Markowitz MV efficient frontiers from the resampling process illustrate the extreme sensitivity of Markowitz MV optimization to estimation error. The many alternative market scenarios produced by the resampling process provide a rich basis for understanding the inherent uncertainty in investment information in the MV optimization process. The thousands of simulations that explore the uncertainty in Markowitz efficient frontiers in practical applications are likely to reflect examples of fat tail, black swan, and other exotic event scenarios.

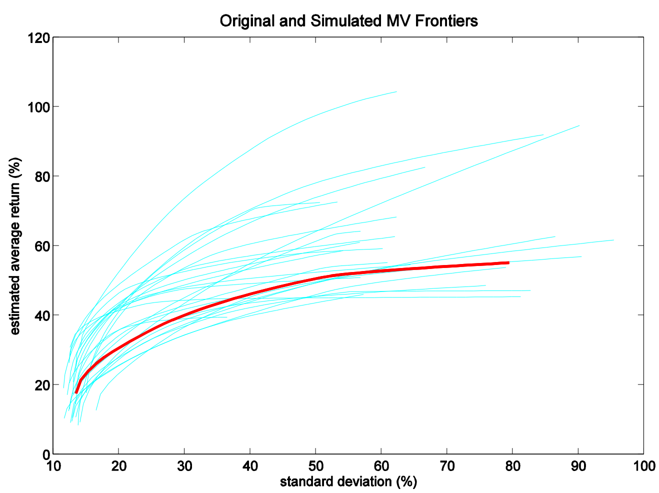


Figure 1: Simulated MV Frontiers from Resampled Inputs

2.0 Efficient Frontier Averaging

The Morningstar and Michaud optimization procedures differ in how they use the information in Figure 1. We present the two procedures below.

2.1 Morningstar Efficient Frontier Averaging Process

The Morningstar procedure is described in Idzorek (2006). The method is similar in many respects to Michaud (1998, Chs. 4, 5).

The simulated frontiers in Figure 1 are MV efficient frontier portfolios displayed relative to their corresponding resampled risk-return inputs. The portfolios on each frontier are selected by arc-length rank.[2] The risk and return of each of the computed portfolios in Figure 1 are recomputed and plotted in

[2] The efficient frontiers displayed in Figure 1 are derived from computing fifty-one Markowitz MV efficient frontier portfolios equally spaced with respect to arc length along the efficient frontier from low to high risk.

terms of the original risk-return estimates. The revised definition of the simulated portfolios' risks and returns in terms of the original estimates implies that each efficient frontier portfolio in Figure 1 must necessarily lie below or on the original Markowitz efficient frontier. Figure 2 reflects the results of plotting the simulated MV efficient frontier portfolios in terms of the original mean-variance inputs.

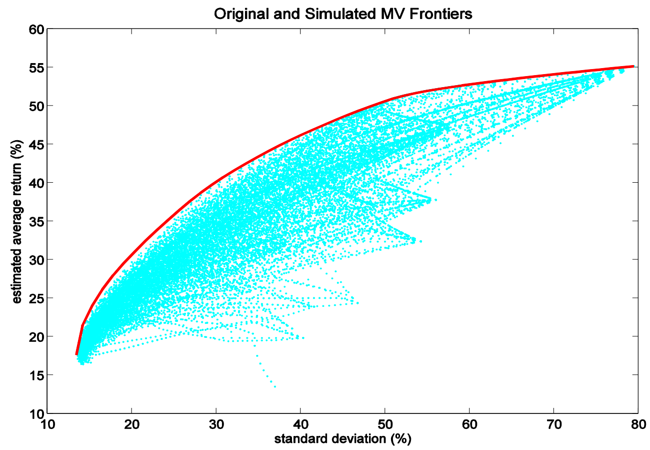


Figure 2: Markowitz and Simulated MV Efficient Portfolios

The Morningstar algorithm proceeds by dividing the risk spectrum spanned by the Markowitz efficient frontier into equal lengths of bins of standard deviations beginning at the minimum variance portfolio and ending at the maximum return portfolio. Each of the cyan simulated efficient frontier portfolios in Figure 2 is assigned a bin with respect to its standard deviation. The resulting Morningstar efficient frontier is computed as the average of the portfolios in each bin and displayed as the risk and return of the average portfolio. Figure 3 displays the efficient frontier associated with the Morningstar process for the Michaud (2008b) data from the Encorr software using default options.

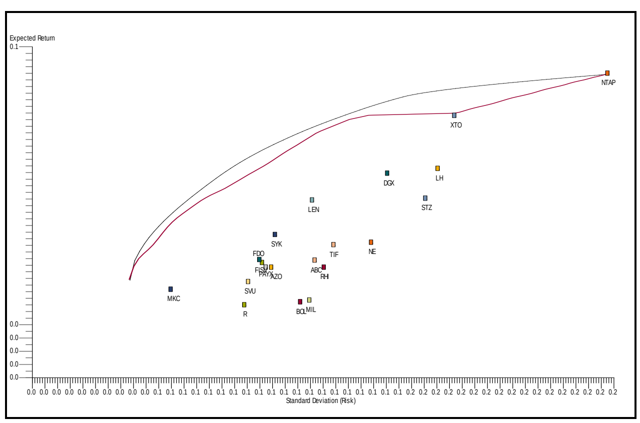


Figure 3: Markowitz and Morningstar Efficient Frontiers

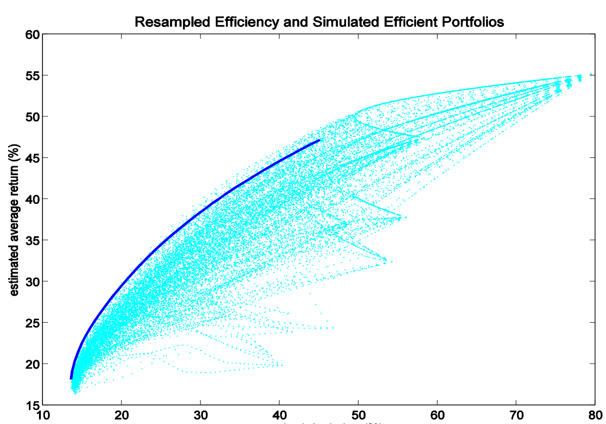
The procedure depends critically on the number of bins used in the Morningstar procedure. Different numbers of bins will result in different portfolios assigned to each bin and will produce different efficient portfolios. Because no theory supports the process, the number of assigned bins is arbitrary and ad hoc. The procedure may produce bumpy and irregular frontiers of noisy portfolios that may include concave and convex segments as in Figure 3. The bins at the high end of the risk spectrum are likely to contain fewer portfolios, resulting in greater Monte Carlo error. By nature of the process the number of portfolios per bin may be very uneven. Predictably, bins which contain a constituent asset's standard deviation are particularly noisy since many portfolios in that bin will have weights at or near 100% for that asset when it has the maximum simulated return. In investment terms, these bumps are likely to cause unnecessary noise-based trading for managers as their risk preferences shift or as the market normally drifts and portfolios need to be rebalanced.

The arbitrariness of the binning process essentially defeats the imperative of addressing the limitations of MV portfolio optimization. Non-uniqueness implies ambiguity and instability, two essential limitations of traditional MV optimization. Moreover, we will show that the portfolios produced by the procedure are often investment unintuitive and not sensibly well diversified.

2.2 The Michaud Efficient Frontier Averaging Process

The Michaud efficient frontier procedure is described in Michaud (1998, Ch. 6) and Michaud and Michaud (2008a, Ch. 6, 2008b).[3] The fundamental principle of the averaging process is derived from expected utility analysis. Consider an investor with minimum risk preferences. Referring to Figure 1, such an investor will choose the minimum variance efficient frontier portfolio in each case of the simulated MV efficient frontiers. By definition each resampled MV efficient frontier is statistically equivalently optimal as any other. There is no reason to choose one frontier over another. The minimum variance portfolio on the Michaud efficient frontier for such an investor is defined as the average of the portfolio weights of all simulated minimum variance efficient frontier portfolios. Similarly, the maximum return Michaud efficient frontier portfolio for an investor with maximum return preferences is defined as the average of all the simulated maximum return MV efficient frontier portfolios. For any other point on the Michaud efficient frontier, consider an investor with a utility function with given risk aversion parameter. Michaud efficient frontier portfolios are an average of all tangent portfolios for a constant utility function parameterized by risk aversion preference. The entire efficient frontier can be computed by spanning the spectrum of risk aversion parameter preferences. By definition, the Michaud efficient frontier is theoretically unique.[4]

Figure 4 illustrates the Michaud efficient frontier for the Michaud (2008b) data set. The simulated efficient portfolios are plotted in terms of the original risk-return inputs. The Michaud frontier displayed within the set of simulated efficient frontier portfolios reflects a process that considers all the many ways things can happen that are consistent with best risk-return estimates.



[3] A rigorous mathematical definition is given in Michaud, Esch, Michaud (2012).

[4] There are more compute-efficient methods for estimation. The return-rank algorithm in Michaud (1998, Ch. 6) and arc-length algorithm in Esch (2012) are faster accurate procedures for estimating the frontier.

Figure 4: Michaud Frontier and Simulated Portfolios

3. Efficient Frontier Composition Maps

The Morningstar optimization process attempts to improve on Markowitz MV portfolio efficiency via resampling. In this section we further analyze the portfolios produced by both the Morningstar and Michaud procedures using portfolio composition maps.

For purposes of comparison, we begin in Figure 5 with the composition map of the portfolios in the Markowitz efficient frontier for the Michaud (2008b) data. The asset allocations for each of the portfolios on the efficient frontier are displayed relative to the twenty color coded allocation to the assets in the data set from the left hand side or minimum variance portfolio to the right hand side or maximum return portfolio. Note that high risk Markowitz efficient frontier portfolios are largely represented by four or fewer assets and may often not reflect perceptions of proper diversification for experienced investors.

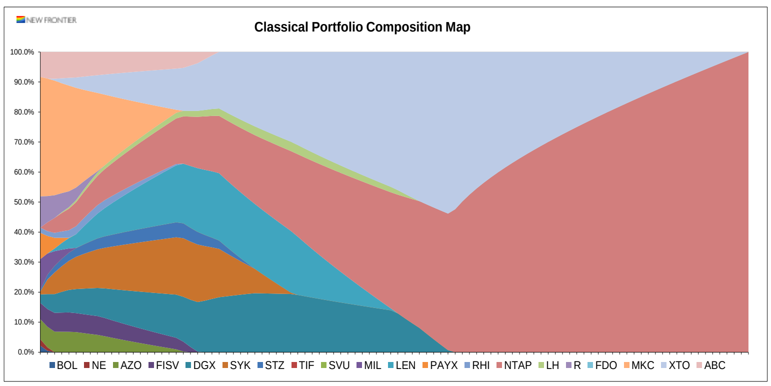


Figure 5: Markowitz Efficient Portfolio Composition Map

3.1 The Morningstar Efficient Frontier Portfolios

Figure 6 provides a composition map of the efficient portfolios defined by the Morningstar procedure for the Michaud (2008b) data.[5] The results can be compared to the Markowitz efficient frontier composition map in Figure 5. While there are differences, notably the inclusion of small allocations for a number of assets at higher levels of efficient frontier risk, the results are surprisingly similar. In particular, as in the Markowitz case, the maximum return efficient portfolio includes only one asset.[6]

Changing the number of bins will change asset allocations across the Morningstar frontier. For example, a 250-bin case will necessarily converge to very different sets of portfolio weights. One reason is that standard deviation bins may arbitrarily include many points from certain simulations but zero points from other simulations in their final optimal portfolio calculations. Consequently each bin assigns its own weights to the Monte Carlo simulations. There is indeed no well-defined or stable probability model for outcome scenarios in the Encorr optimization procedure. By contrast, the Michaud procedure gives every scenario an equal weight in the calculation of the resampled frontier, and corresponds to a consistent well-defined statistical model for asset returns across all parts of the resampled frontier, regardless of the number of frontier points requested in the analysis.

[5] In the Encorr software a composition map is called a "Frontier Area Graph."
[6] The Encorr software provides a cosmetic option to "smooth" out the jagged edges that may be observed in the composition map resulting from the Morningstar procedure. See the appendix for illustration and further discussion.

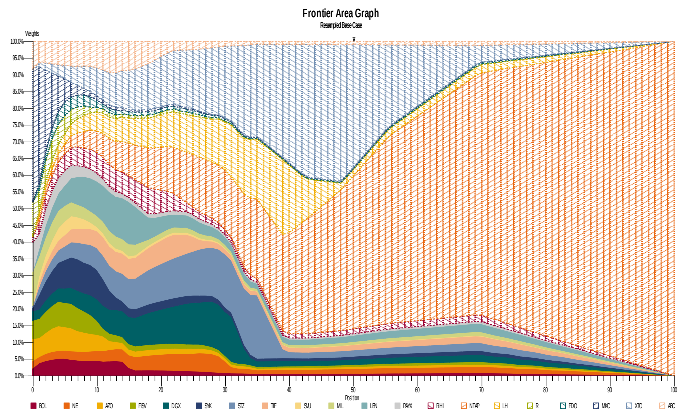


Figure 6: Morningstar Frontier Composition Map: Encorr software. 50 bins.

3.2 Michaud Efficient Frontier Composition Map

Figure 7 provides a composition map of the efficient frontier portfolios associated with the Michaud procedure for the Michaud (2008b) data set in Figure 4. The results can be compared to those in the composition map in Figures 6 and 7 for the Morningstar frontier portfolios. The contrast in the results for the two procedures is stark. The display shows superior investment diversification and smooth transitions along the risk spectrum across the entire frontier. No further processing is required to smooth out the results or present the findings for cosmetic objectives. Note that even the maximum return portfolio on the Michaud efficient frontier is well diversified. The procedure is stable, produces investment intuitive portfolios, and is theoretically unique.

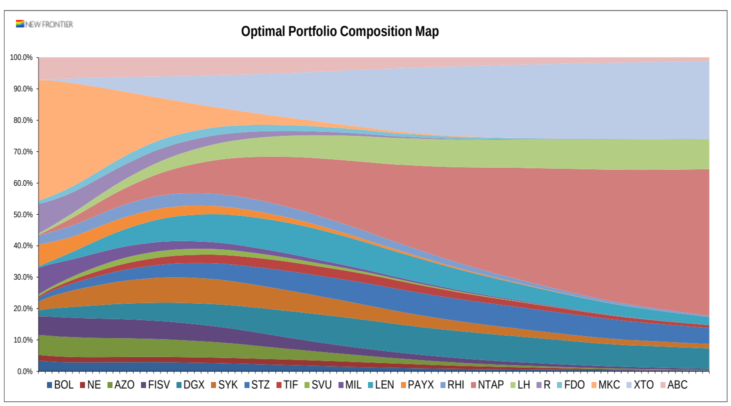


Figure 7: Portfolio Composition Map from New Frontier's software

It may be of interest to further analyze the efficient frontier maximum return portfolios in the Morningstar and Michaud frontier cases. Figure 8 below provides a detailed illustration in pie chart form for the two procedures. While the Morningstar portfolio has 100% of one asset, the Michaud portfolio is a blend of many assets.

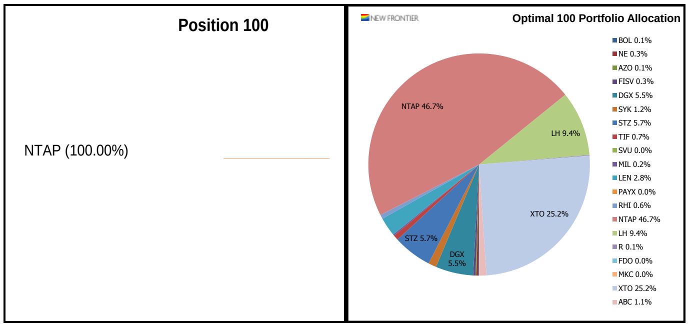


Figure 8: Morningstar and Michaud maximum return efficient portfolios

4.0 Summary and Conclusion

While theoretically robust under a wide range of assumptions, Markowitz MV efficiency has serious limitations for practical management due to estimation error sensitivity. These include ambiguity, instability, and poor out-of-sample investment characteristics.

Morningstar optimization is an attempt to improve on Markowitz MV efficiency by introducing resampling of risk-return estimates while avoiding the commercial limitation of licensing the superior but patented Michaud optimization procedure. Their solution introduces a binning process associating in-sample standard deviations of resampled efficient portfolios that span the spectrum of risk of the Markowitz efficient frontier. The results define a different efficient frontier of portfolios. We illustrate the procedure with a twenty asset case based on twenty years of monthly historical returns. The efficient frontier illustrated in Figure 3 may often have a bumpy non-investment intuitive character.

The binning process introduced by Morningstar is not grounded in financial or econometric theory. The ad hoc character of the procedure leads to non-unique solutions that depend critically on the definition of the number of bins. Different bin definitions will lead to different Morningstar frontiers and different efficient portfolios at given risk levels. The frontiers can have many undesirable properties including multiple convex and concave segments. In contrast, Michaud optimization illustrated in Figure 4 is theoretically unique.[7]

The composition maps of Morningstar frontier portfolios display further limitations of the procedure. The resulting portfolios may often exhibit poor diversification and awkward transitions along the risk spectrum between adjacent portfolios. The Encorr smoothing process serves only to improve the appearance of the composition maps and is in no way an enhancement of portfolio optimality.[8] The procedure is rooted neither in financial theory or sound statistical principles and may often lead to poorly constructed portfolios that appear to provide little improvement over Markowitz efficiency. In addition, no simulation tests have been offered to support enhanced out-of-sample investment performance.

In contrast, Michaud optimization has been the subject of rigorous simulation studies that show that the procedure is likely to enhance investment performance on average.[9] In addition New Frontier's

[7] While there are alternative methods for creating the Michaud frontier, the choice is dependent on the notion of compute-efficiency.
[8] See the Appendix A for further discussion.
[9] Michaud (1998, Ch. 6), Michaud and Michaud (2008a, Ch. 6, 2008b).

resampling process has been tested in Markowitz and Usmen (2003) simulation studies under challenging conditions.[10] Without recourse to artifice, Michaud optimization creates a smooth and sensible composition map of well diversified portfolios across the risk spectrum. Its approach can be rationalized with statistical science as a best guess for the unknown true optimal frontier, given the information available in the inputs.

The non-uniqueness of the Morningstar procedure fundamentally defeats the imperative of addressing the limitations of Markowitz MV efficiency in practical application. This is because the Morningstar procedure represents ambiguous optimality, an unstable framework, and investment unintuitive solutions. In practice analysts are likely to require many ad hoc constraints and revisions of risk-return estimates to provide acceptable solutions for marketing purposes. The "why bother optimizing" problem of Markowitz optimization remains unsolved because such measures are necessary. There is no foundational reason for likely improved investment performance. The Morningstar optimization agenda of superior performance relative to Markowitz and as a fiduciary alternative to Michaud optimization is unfulfilled.

Appendix A:
Morningstar Smoothing

In order to reduce the irregularities in the frontier portfolios, the Encorr optimizer can optionally smooth the portfolio weights. An example of a smoothed version of our example case appears in Figure 9. The results of the smoothing procedure can be compared to the 250 bin case of Figure 6. The method may somewhat mitigate the appearance of wobbly irregularities in the composition map but is unrelated to optimization and is not designed to improve out-of-sample performance. In the example the smoothing eliminates some of the smaller scale instability of the portfolio weights, but preserves most if not all of the undesirable and strange larger scale irregularity of the portfolio weights. Smoothing and optimization are mathematical procedures with different aims. Optimization techniques should result directly in optimal portfolios and not need additional processing to cover flaws. Encorr's smoothing is simply a cosmetic fix to a basic weakness in its approach to portfolio construction.

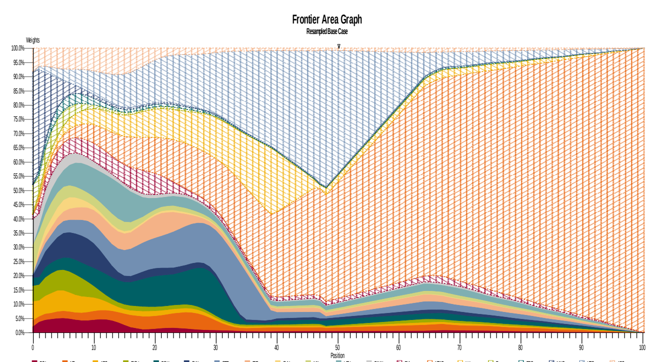


Figure 9: Smoothed Encorr Area Graph, analogous to Figure 6.

[10] Michaud optimized portfolios outperformed in simulation tests relative to Markowitz with the handicap of inferior risk-return estimates.

Appendix B:
Twenty Stock Data Set

Standard & Poor's 500 stocks and market capitalizations were taken as of October 2006. The market capitalizations were discounted back via monthly total returns to approximate market capitalizations in January 1997. The 20 Large Cap stock set was taken from a random sample of the 100 largest of these market capitalization stocks. Ten years of complete monthly returns for the sample spans January 1997 through December 2006.

Ticker	Asset Name	Return	Standard Deviation
BOL	Bausch & Lomb	13.5%	36.9%
NE	Noble Corporation	24.4%	46.7%
AZO	AutoZone Inc.	19.9%	33.0%
FISV	Fiserv Inc.	20.7%	31.7%
DGX	Quest Diagnostics	36.9%	49.0%
SYK	Stryker Corp.	25.8%	33.5%
STZ	Constellation Brands	32.4%	54.2%
TIF	Tiffany & Co.	23.9%	41.6%
SVU	SUPERVALU Inc.	17.2%	29.9%
MIL	Millipore Corp.	13.9%	38.2%
LEN	Lennar Corp.	32.0%	38.6%
PAYX	Paychex Inc.	19.9%	32.3%
RHI	Robert Half International	19.9%	40.3%
NTAP	Network Appliance	55.2%	79.4%
LH	Laboratory Corp. of America Holdings	37.8%	56.0%
R	Ryder System	13.1%	29.3%
FDO	Family Dollar Stores	21.2%	31.3%
MKC	McCormick & Co.	15.9%	19.3%
XTO	XTO Energy Inc.	47.5%	58.4%
ABC	Amerisourcebergen Corp.	21.1%	38.9%

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