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Equity Optimization Issues V: Monte Carlo and Optimization Errors

by

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Abstract

Improvements in optimization design and resolutions of fallacies in asset management practice are largely due to recent applications of Monte Carlo simulation technology.

This report is the fifth in a series on proper mean-variance (MV) equity portfolio optimization design and practice in the light of estimation error. Improvements in optimization design and resolutions of fallacies in asset management practice are largely due to recent applications of Monte Carlo simulation technology. This paper focuses on three widespread fallacies in current practice: 1) estimation error in risk can be ignored; 2) all optimization constraints reduce optimized portfolio value; 3) robustness without effectiveness is a useful optimizer characteristic. All three issues have severe negative performance consequences. Indeed, the inverse of recommended practice is often the correct and effective one.

The road map for the paper is as follows. The first section provides a brief review of Monte Carlo simulation and its application to Markowitz (1959) MV optimized portfolio studies. The second section addresses the fallacy that estimation error in risk can be ignored relative to the means (Chopra and Ziemba 1993) (CZ). The third section addresses the invalidity of the "transfer coefficient" as a useful investment tool and the fallacy that constraints per se cause limitations in MV optimized portfolio performance (Clarke, deSilva and Thorley 2002) (CST). The fourth section discusses the investment limitations of robust optimization as a useful optimizer characteristic (Ceria and Stubbs 2005, Feldman 2003). The final section provides concluding comments.

1.0 Monte Carlo Simulation

Monte Carlo simulation is arguably one of a handful of the most important technologies developed during the 20th century. Simulation has had a major impact on the resolution of many important practical problems. During the early days of the Manhattan project, the mathematician Ulam noted that weapons testing and design could be accomplished inside a computer with Monte Carlo simulation. Today the United States no longer does actual atomic weapons testing and research and designs are now tested with Monte Carlo simulation, highly advanced physics and computer technology.

Monte Carlo simulation represents a watershed technology in the ability to understand and properly design MV optimized portfolios.[1] Investment information is endemically uncertain. Simulation studies show that estimation error is a first order factor in MV optimized portfolio performance.[2] Resampled Efficiency™ (RE) optimization, a generalization of MV optimization, properly uses estimation error in defining portfolio

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[1] The difference between previous methods and simulation studies can be described as the difference between "what should be" vs. "what is" optimal. Simulation provides a means of verifying optimized portfolio performance.

[2] Some managers persist in the illusion that "back tests" with historical return data provide reliable information on the future performance of various optimization investment strategies. Paper portfolios do not perform like real portfolios and results, to the extent they are significant, are period dependent. Monte Carlo simulation technology, properly employed, can reliably evaluate optimization design factors.

optimality.[3] An RE optimizer represents a Levy-Markowitz (1979) expected utility maximizing investor in the context of estimation error.[4]

Monte Carlo simulation studies enable understanding of the many ways assets and markets may perform consistent with our forecasts. Typically investors forecast the risk and return for equity indices greater than bonds. But such forecasts reflect much uncertainty. As the number of assets grows, uncertainty compounds relative to defining an optimal mix of securities for a given level of risk. Monte Carlo simulation provides a bridge for understanding how forecast uncertainty affects optimality and likely performance. It also provides a framework for reliably comparing the investment value of one optimization procedure relative to another and for defining more effective optimality procedures in the context of estimation error.

Fundamental technological innovations enable effective research but also relegate quack theories and much conventional wisdom to the dustbin of history. While RE optimization may have a dramatic effect on sensible portfolio structure, ease of use, and improved performance, it will be adversely affected by the many errors that characterize current equity portfolio optimization practice. Quantitative equity managers need to unlearn many invalid notions associated with current optimization practice in order to improve investment performance.

1.1 MV Optimization Simulation Study Framework

A MV portfolio optimization simulation study assumes a "referee" who knows the true value of the MV optimization inputs – means, standard deviations, and correlations (means and covariances) – for a given set of securities. The referee does not tell us the true values. The referee provides (Monte Carlo) simulated returns for the securities that are statistically consistent with the true input values.[5] Optimization inputs are computed from the simulated security returns provided by the referee. The analyst then computes the associated MV optimal portfolios along the Markowitz MV efficient frontier. The referee scores the risk and return of our computed "optimal" portfolios to determine how well the MV optimization process estimated true optimality. This procedure is repeated many times. The average of the true risks and estimated returns of the MV optimized portfolios are computed and compared to true optimality.

1.2 Comparing Optimization Design and Performance

Four classic Monte Carlo MV optimization simulation studies are described below.

1. Jobson and Korkie (1981) compared the investment performance of unconstrained MV optimization vs. equal weighting. They showed that unconstrained MV

[3] RE optimization was invented by Richard Michaud and Robert Michaud and is a U.S patented procedure, worldwide patents pending. It was originally described in Michaud (1998, Ch. 6). New Frontier Advisors, LLC (NFA) is exclusive worldwide licensee.
[4] It is beyond the scope or purpose of this report to detail why RE optimization is actually consistent with Levy-Markowitz expected utility maximization investing. We will return to this issue in future reports. This issue is one of the more important sources of misunderstandings of RE optimization.
[5] In statistical parlance, the true values are population parameters and the means, standard deviations, and correlations computed from the simulated returns are the sample statistics.

optimization was highly affected by estimation error in risk and return and the optimized portfolios had, on average, little if any investment value. An equal weighted portfolio provided far superior performance.

2. Frost and Savarino (1988) compared the investment performance of unconstrained vs. sign constrained MV optimization similar to the Jobson and Korkie framework. They showed that sign-constraints substantially improved investment performance.

3. Michaud (1998, Ch. 6) compared RE vs. MV optimized portfolios. He showed that RE optimization substantially improved risk-adjusted performance.

4. Markowitz and Usmen (2003) compared MV optimized portfolio and diffuse Bayes input estimation vs. RE optimization with unimproved inputs. They showed that RE optimization with unimproved inputs provided superior performance relative to MV optimization with improved inputs.

Each of these studies provides reliable information that sharply contradicts much conventional wisdom and current practice on optimizing portfolios.[6]

2.0 Importance of Risk Estimation Error

Chopra and Ziemba (1993) (CZ) have been a primary reference for the notion that estimation error in risk can be ignored relative to the means. Their paper examined the relative impact of estimation error in means, variances, and correlations on MV optimization. For a representative case, they claimed that estimation error in the means is more important, by a factor of 11, relative to variances, and estimation error in the variances is twice as important as correlations. As we will show, the CZ claim is fallacious. Unfortunately, the CZ fallacy has been the rationale for many errors in optimization design and has likely often been the cause of poor investment performance in practice.

2.1 A Flawed Framework

The procedure used by CZ to examine estimation error is not a Monte Carlo simulation study. It is a perturbation study that depends on a particular utility function. Varying the mean, variance, and correlation parameters changes the value of the utility function. They compare the cash equivalent (CE) of the utilities to determine how close the utility function with errors is to the true utility value.

The framework is flawed because the measure of differences relative to estimation error is only in terms of changes in CE utility value. There is no measure of how the portfolios with estimation error actually performed.[7] Such a measure can only be estimated with

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[6] Our series of newsletter research articles on MV equity portfolio optimization, available at http://www.newfrontieradvisors.com addresses many additional issues in this area. Note that the Markowitz and Usmen result implies that RE optimization, rather than improved input estimation, may be the first order factor for improved portfolio performance.

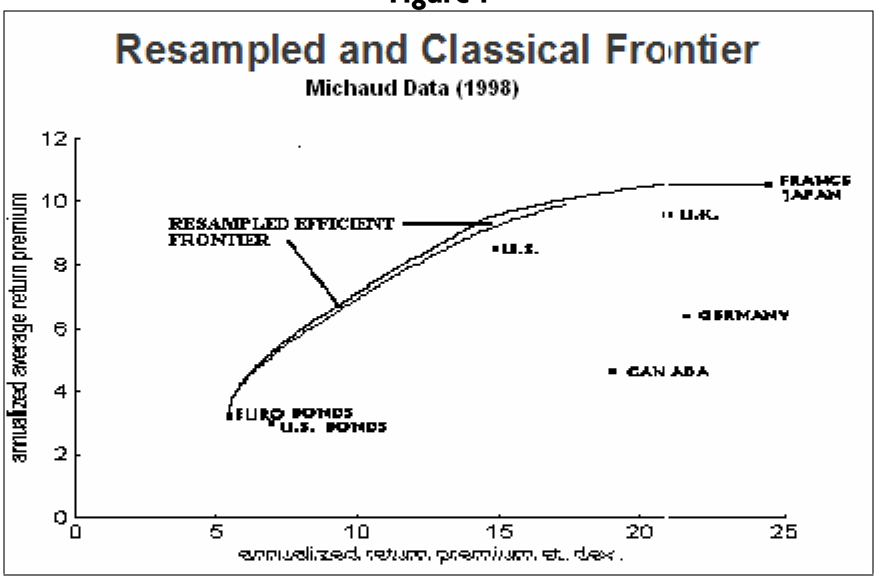
[7] In statistical parlance, CZ is a study of in-sample estimation error rather than out-of-sample investment performance.

Monte Carlo simulation. Their research conclusions have no reliability for actual investment performance.

Figure 1 below presents the in-sample MV and RE optimization efficient frontiers for the data in Michaud (1998). Note that the RE efficient frontier in-sample is below the MV efficient frontier. Any traditional utility function analysis as in CZ would indicate that the RE optimal portfolios have less "utility" than the MV optimized portfolios for a given level of risk.[8] However, we know from Monte Carlo simulation studies that the RE optimal portfolios outperform the MV optimal portfolios on average in the investment period. Few rational investors will prefer improved (in-sample) utility to improved (out-of-sample) investment performance.[9]

As noted in the introduction, Monte Carlo simulation studies invalidate many claims and much earlier research. The CZ results are in-sample comparisons that can make no claim to reliable investment results ex post. Our critique has wide applicability for the invalidity of a long list of results in textbooks and published articles on portfolio optimization.

Figure 1



2.2. A Simple Counter-example

Beyond the fact of a flawed framework, there is much obvious evidence to contradict the CZ claim. Using simulation studies, Jorion (1996) found that estimation error in the covariance matrix was often dominant over the means. One simple reason is that, as the number of assets grows, estimation error in the means grows linearly while estimation error in the covariance matrix grows quadratically. Since equity portfolio optimization

[8] The Harvey et al (2003) critique of RE optimization is based on the same invalid in-sample utility argument used in CZ.

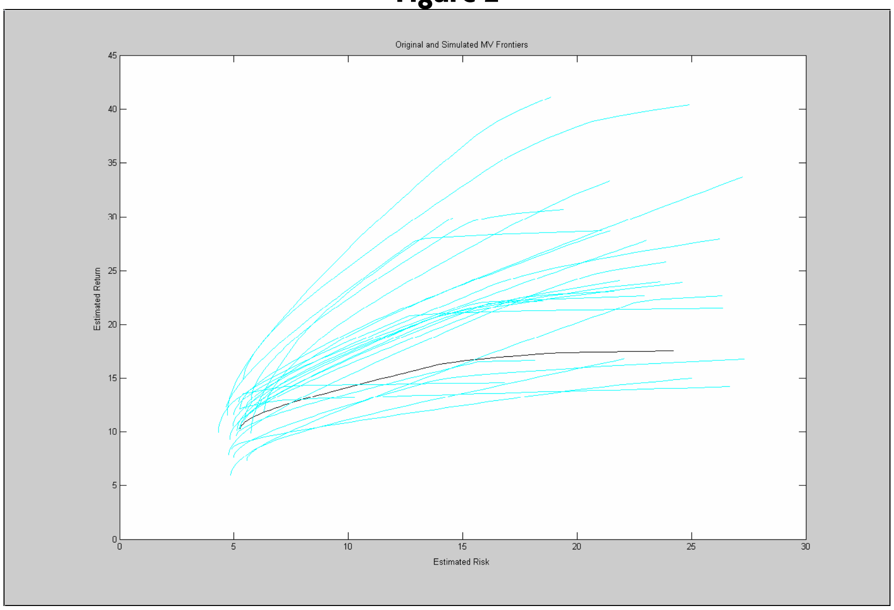
[9] Harvey et al (2003) appear to define rational investor behavior as a preference for in-sample utils over out-of-sample improved risk-adjusted performance.

may involve thousands of securities, estimation error in risk will often dominate estimation error in the means in practice.

2.3. Visual Analysis

The CZ error is apparent in any simulation of optimized portfolio behavior even for small optimization universes with highly diversified asset classes. Figure 2 below presents twenty-five Monte Carlo simulated MV efficient frontiers based on historical return data for eight capital market indices (Michaud 1998). The black frontier in the middle of the cyan MV efficient frontiers represents the traditional MV efficient frontier. The cyan efficient frontiers are all simulated with resampling (or bootstrapping) from the original eighteen years of monthly return data.

Figure 2



Examination of the simulated MV efficient frontiers shows that they are far from being equally risky even in this case where risk estimation error is relatively minimal. Some simulated frontiers extend far less, and others far more, than the risk of the original (black) MV efficient frontier. Resampling (or bootstrapping) only the returns while leaving risk constant would have ignored important estimation error consequences in risk in this simple case.[10] Equity portfolio optimization simulations would have many more assets with far more variance than that shown with these highly diversified indices.[11]

2.4 Equity Portfolio Risk Estimation Implications

The notion of estimation error in risk not being material is most evidently in error when seen from the perspective of risk estimation for equity portfolio optimization. MV

[10] As recommended by Feldman (2003).
[11] Individual stock risk, for example, is on average roughly twice as large as diversified equity capital market indices.

optimization requires a well conditioned covariance matrix whatever the model for portfolio risk. Consider that an equity portfolio optimization with respect to the S&P 500 stock index requires covariance estimation for a minimum of 500 stocks. A well conditioned covariance matrix requires a minimum of 501 (N+1) independent observations.[12] But commercial risk models are typically estimated with 60 months of historical return data, far fewer than the number of periods required for minimal use in a MV optimization.[13] While equity risk models may serve their primary purpose of portfolio risk analysis, their estimation error properties for portfolio optimization are severe. Econometric methods are available to impose conditioning on the equity risk model covariance matrix for optimization purposes. However, such methods do not eliminate the enormous estimation error implicit in the risk model estimation process. The estimation error issue necessarily compounds for larger benchmark optimizations. From this perspective it is clear that resampling is absolutely essential for minimizing the effect of estimation error on MV equity portfolio optimization.

2.5 An Intuitive Argument

As Figure 1 shows, the Resampled Efficient Frontier™ (REF) does not allow the investor to take as much risk as the MV efficient frontier.[14] Why is risk taking limited? Is the REF behavior investment intuitive? MV optimization assumes 100% certainty in the input estimates. If you are 100% certain of your information you are willing to put more money at risk. If you are less than 100% certain in your information you are willing to put less money at risk. REF avoids taking as much risk as the MV efficient frontier because it is less certain of the information it has to make investment decisions. This example provides an illustration of RE optimization's consistency with fundamental principles of rational decision-making under uncertainty. The implication is that an optimizer that does not resample risk as well as return poorly represents rational investor behavior.[15]

2.6 Summary

From a variety of points of view, estimation error in risk is as important as in the means. Is estimating rankings of assets more error prone than relationships among assets? Estimation error in risk as well as return must both be addressed in order to compute investment useful optimized portfolios. Any method that addresses estimation error in return only is unlikely to have more than cosmetic value.

3.0 Constraints in Optimized Portfolio Performance

Clarke, deSilva, and Thorley (1992) (CST) define the superficially appealing concept of a "transfer coefficient" (TC) to show that optimizer performance is adversely affected by optimization constraints. CST use TC to rationalize the empirically observed fact that optimized portfolios often don't outperform their benchmarks in spite of significant levels of investor information. They propose that removing constraints improves MV optimized portfolio performance.

[12] Muirhead (1982) p. 82.
[13] Some commercial risk models are estimated on weekly returns over a three year period.
[14] There are situations where this is not the case but involve technical issues beyond the scope of this report.
[15] Such approaches also often have substantial statistical estimation instabilities as well.

3.1 The Transfer Coefficient

CST defines TC as the correlation between traditional or constrained MV optimized portfolio weights and ex ante alpha. CST claims that TC provides a measure of the negative effect of optimizer constraints on optimized portfolio performance. Assume a statistically significant IC or "information correlation."[16] The IC is a measure of the "signal quality" in the forecasts. According to CST analysis, if there are no optimization constraints, TC equals one and observed optimized portfolio performance is a direct function of IC. Since in practice MV optimized portfolio weights are typically constrained, TC is a measure of how much information is transferred to the optimized portfolio by the optimization. The notion is that a higher TC indicates likely better performance and the implication is that optimizers should be as minimally unconstrained as possible.

3.2 TC Analysis is Invalid

To validate their interpretation of TC, CST has to prove that unconstrained optimized portfolio weights are a direct function of IC. This is formula (5) in CST: optimized unconstrained portfolio weight equals the risk-weighted forecasted alpha times a constant. Formula (5) is required to validate TC.

CST requires three assumptions to prove formula (5): 1) Validity of the Grinold (1989) formula for the information ratio of a MV optimized portfolio. 2) Diagonal covariance matrix for residual returns. 3) No budget constraint imposed on the optimization process. Each of these assumptions has serious limitations for optimized portfolio optimization in practice.

Michaud and Michaud (July 2005) (MM) show that the Grinold (1989) formula is invalid when applied to asset management practice. In fact, as MM shows, the prescriptions associated with the Grinold formula are not only generally invalid but perverse in that they recommend the opposite of what a manager should do.[17] The CST assumption of a diagonal covariance matrix, also discussed in MM, is highly restrictive and is, in general, invalid for most institutional equity portfolio optimization in practice. The CST assumption of no budget constraint implies the absurd situation where investment in any asset is unlimited. In actual investment practice, investment in an asset generally limits investment in others. The bottom line is that the claimed relationship between TC and unconstrained MV optimized portfolio weights is invalid. TC has no obvious investment value and is not a measure of the information lost in a constrained MV optimized portfolio.

3.3 A Simple Counter-example

Whatever the limitations of CST analysis, what do we make of the claim that constraints limit the investment value of MV optimized portfolios? As indicated earlier, the Frost and Savarino (1988) simulation studies showed that, contrary to CST, out-of-sample

[16] The IC is sometimes called the "information coefficient." It is the correlation between ex ante and ex post return. In equity portfolio optimization, return is alpha or residual risk-adjusted return. There are a number of variations of the definition of alpha in practice but these differences are not material in this context.
[17] We refer to the invalid recommendations in Grinold and Kahn (1995, Ch. 6, p. 130).

performance improved with sign constraints. In general, financially meaningful optimization constraints are your friends.

3.4 A Paradox Explained

If the CST TC analysis is invalid, what accounts for the relative lack of ex post performance of MV optimized portfolios when IC is significant? The obvious answer, as demonstrated by Monte Carlo simulation studies, is estimation error not portfolio optimization constraints. In fact, financially meaningful constraints tend to improve MV optimized portfolio performance.

3.5 Postscript

Jobson and Korkie's (1981) classic studies showed that unconstrained MV optimized portfolios have little, if any, investment value on an absolute scale or relative to equal weighting. In fact, it can be argued that CST's "constraint-induced noise" coefficient derived from formula (A21) is actually the bearer not of noise as they claim but of much of the valuable information in the optimization process.

4. Robust vs. Effective Optimization

RE optimization is both a robust and investment effective optimizer. It is robust because small changes in estimates of risk and return typically result in small changes in the optimized portfolio weights. It is also provably effective at improving the investment value of MV optimized portfolios (Michaud 1998, Ch. 6). Classical MV optimization is neither robust nor investment effective. RE optimizer robustness comes from the resampling and averaging process. Resampling is a statistical method for deriving more information from data. RE optimization does not change investment information but uses it in a far more investment meaningful way for defining optimality.[18]

Robustness is sometimes claimed to be a desirable feature of an optimizer. But robustness without effectiveness is undesirable. Two recent proposals for robust optimization discussed below are strongly affected by the CZ fallacy and have significant practical investment limitations.

4.1 Robust Optimization with Shrunk Alphas

Ceria and Stubbs (2005) (CS) propose a MV equity optimizer that heuristically shrinks stock alpha. The CS proposal creates MV optimizer robustness by arbitrarily ignoring much of the manager's information. In practice, institutional stock alphas are typically defined to have a very narrow range of values to limit the arbitrary behavior of the MV optimization process.[19] The investment consequences of shrinking already shrunk alphas should give managers pause to consider the investment implications of the CS procedure. In contrast,

[18] Improved inputs may also improve the investment value of optimized portfolios. As reviewed in Michaud (1998, Ch. 8), Stein estimators reduce estimation error and often improve performance. The Bayes procedure in Markowitz and Usmen (2003) is another example of a rigorous statistical procedure for reducing estimation error and improving likely performance.

[19] Institutional asset managers often define alpha to have a range of plus or minus 3% or less.

RE optimization creates robustness without the need to change a manager's forecasts.[20] Note also that the CS optimized portfolio is effectively a MV optimized portfolio that inherits associated ex post MV optimization performance limitations.

4.2 Robust Optimizer with Resampled Means

Feldman (2003) proposes a robust MV optimizer based on resampling and averaging means but not risk. Earlier discussion indicated the limitations of ignoring estimation error in risk. Such a procedure has unproven investment value and significant statistical instability estimation limitations.[21] It also does not validly represent rational investor risk behavior in the context of uncertainty.[22]

Conclusion

Many invalid procedures characterize current equity portfolio optimization in practice. We addressed three fallacies: 1) risk estimation error can be ignored; 2) optimization constraints are generally the cause of poor optimization performance; 3) robustness without effectiveness is a desirable characteristic of portfolio optimizers. These errors not only limit but often perversely affect the potential for improved optimized portfolio performance. A statistical understanding of portfolio optimization and investment information is the key to developing valid and effective practices. Monte Carlo MV optimization simulation studies provide a watershed technology for reliably improving investment value and shedding invalid conventional wisdom. Much underbrush of misguided practices needs to be cleared in order to fulfill the Markowitz optimization promise of improved investment performance.

[20] Note that there is no conflict between RE optimization and input estimation procedures that reduce estimation error. The point is that the RE optimization robustness property is not a function of ignoring information but of using information effectively.

[21] The binning in the Feldman procedure is a highly arbitrary unstable estimation process.

[22] In Feldman's proposal, unlike RE optimization, the range of risky efficient portfolios is unaffected by information uncertainty.

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